

# **Propagation and neutrino oscillations in the base of a highly magnetized gamma-ray burst fireball flow**

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## ABSTRACT

Neutrons play an important role in the dynamics of gamma-ray bursts. The presence of neutrons in the baryon-loaded fireball is expected. If the neutrons abundance is comparable to that of protons, important features may be observed such as quasi-thermal multi-GeV neutrinos in coincidence with a subphotospheric  $\gamma$ -ray emission, nucleosynthesis at later times and rebrightening of the afterglow emission. Additionally, thermal MeV neutrinos are created by electron-positron annihilation, electron (positron) capture on protons (neutrons) and nucleonic bremsstrahlung. Although MeV neutrinos are difficult to detect, quasi-thermal GeV neutrinos are expected in  $\text{km}^3$  detectors and/or DeepCore+IceCube. In this paper, we show that neutrino oscillations have outstanding implications for the dynamics of the fireball evolution and also that they can be detected through their flavor ratio on Earth. For that we derive the resonance and charged-neutrality conditions as well as the neutrino self-energy and effective potential up to order  $m_W^{-4}$  at strong, moderate and weak magnetic field approximation to constrain the dynamics of the fireball. We found important implications: i) resonant oscillations are suppressed for high baryon densities as well as neutrons abundance larger than that of protons and ii) the effect of magnetic field is to decrease the proton-to-neutron ratio aside from the number of multi-GeV neutrinos expected in DeepCore detector. Also we estimate the GeV neutrino flavor ratios along the jet and on Earth.

*Subject headings:* Physical Data and Processes: magnetic fields — Physical Data and Processes: neutrinos — Stars:gamma-ray burst: general — stars: jets

## 1. Introduction

Gamma-ray bursts (GRBs) are brief events occurring at an average rate of some days throughout the universe. These short, energetic bursts of gamma-rays mark the most violent, cataclysmic explosions in the universe, likely associated with the birth of stellar-size black-holes or rapidly spinning, highly magnetized neutron stars. One of the most successful theory in terms of explaining GRBs and their afterglows is the fireball model (see Mészáros 2006; Zhang & Mészáros 2004, for recent reviews). This model predicts an expanding ultrarelativistic shell that moves into the external surrounding medium. The collision of the expanding shell with another shell (internal shocks) or the interstellar medium (external shocks) gives rise to radiation emission through the leptonic (synchrotron and synchrotron self-Compton (SSC) radiation) or hadronic (photo-pion decay and inelastic proton-neutron collision) processes. In addition, when the expanding relativistic shell encounters the external medium two shocks are involved: an outgoing, or forward shock (Rees & Mészáros 1994; Paczyński & Rhoads 1993) and another one that propagates back into the ejecta, the reverse shock (Mészáros & Rees 1994, 1997a).

The base of the fireball flow is connected to the GRB central engine, a black hole (BH) - torus system or a rapidly rotating magnetar. It is endowed with magnetic fields and formed by free nucleons,  $e^\pm$  pairs in thermal equilibrium of 1 – 10 MeV inside the initial scale  $\sim 10^7$  cm assuming a  $10M_\odot$  BH. In the initial state, the baryons are essentially at rest with respect to the central engine (Zhang & Mészáros 2004).

In recent years, two observational facts have supported the idea that the central engine could be magnetized. i) The strong gamma-ray polarization (Coburn and Boggs 2003; Boggs & Coburn 2003; Rutledge & Fox 2004) and ii) a stronger magnetic field in reverse than in forward shock (magnetization of the jet) as a result of a full description of GRBs 980923, 990123, 021004, 021211, 041219A, 090926A and others (Zhang & Meszaros 2002; Zhang et al. 2003a; Fan et al. 2005; Fraija et al. 2012a,b; Sacahui et al. 2012), so that the fireball may be endowed with a primordial magnetic field (Zhang et al. 2003a; Zhang & Kobayashi 2005). Recently, a relevant

result based on the magnetization of outflow and the non detection of neutrinos from GRBs by IceCube (Abbasi & et al. 2012b) was found by Zhang & Kumar (2013). They proposed that the suppression of the neutrino flux in the gamma-ray emission region is a consequence of the degree of magnetization of the outflow.

The evolution of the fireball with magnetic fields has been explored by various authors (Usov 1999; Wheeler et al. 2000; Blandford 2002; Lyutikov & Blandford 2002; Spruit & Drenkhahn 2003; Vlahakis and Königl 2003a,b; Zhang & Kumar 2011). In these models an electromagnetic component is introduced in the dynamics of the standard fireball. This extra component is taken into account through the magnetization parameter ( $\sigma$ ), defined as the ratio of Poynting flux (electromagnetic component) to matter energy (internal+kinetic component). Depending on the dissipation process, a part of the dissipated magnetic energy is converted into internal energy, and the other part is used to accelerate the fireball (Michel 1969; Mészáros 2012).

The energy requirement (isotropy-equivalent luminosities  $L_\gamma \geq 10^{52} \text{ erg s}^{-1}$ ) demands magnetic fields at the base in excess of  $B \sim 10^{15} \text{ G}$ , that can be produced by shear and instabilities in an accreting torus around the BH. The energy source can be either the accretion energy or via magnetic coupling between the disk and BH, extraction of angular momentum from the latter occurring via the Blandford-Znajek mechanism (Blandford & Znajek 1977; Mészáros & Rees 1997b; Usov 1992; Lee et al. 2000; Putten et al. 2001).

On the other hand, the presence of neutrons and their abundance in the fireball GRB plays an important role in the dynamics of the jet and its relation with the observed photon and neutrino flux emission (Koers & Giannios 2007; Metzger et al. 2008; Razzaque & Meszaros 2006a; Dermer & Atoyan 2006). Among the more relevant consequences are: i) generation of quasi-thermal GeV neutrinos and subphotospheric  $\gamma$ -ray emission via inelastic collisions between streaming protons and neutrons in the fireball. Above the pion production threshold ( $\sim 140 \text{ MeV}$ ), the n-p decoupling is followed by inelastic n-p scattering leading to  $\pi^\pm$  and  $\pi^0$ . As known,  $\pi^0$  decays to two photons ( $\gamma + \gamma$ ) and  $\pi^\pm$  to four particles (three neutrinos) ( $\mu^+ + \nu_\mu$  and then  $\mu^+$  to  $e^+ + \nu_e + \bar{\nu}_\mu$ ), which

is the first signature in photons and neutrinos (Derishev, et al. 1999; Bahcall & Meszaros 2000; Meszaros & Rees 2000; Razzaque & Meszaros 2006b; Murase et al. 2013), ii) the effects on the development, strength and energy range of internal shocks (Rossi et al. 2006; Xue et al. 2008; Fan et al. 2005; Fan & Wei 2004), iii) nucleosynthesis (Metzger et al. 2008; Beloborodov 2003a), and iv) rebrightening of the afterglow emission. Neutrons decay after 881.5 s, hence they play an important role at large radii  $\sim 10^{16} - 10^{17}$  cm where external shocks happen as even an exponential small number of survival neutrons carry an energy much larger than the rest-energy of circumburst medium (Derishev, et al. 1999; Beloborodov 2003a,b; Rossi et al. 2006).

Furthermore, in the initial stage and during the expansion of the fireball thermal neutrinos will be produced by electron-positron annihilation ( $e^+ + e^- \rightarrow Z \rightarrow \nu_j + \bar{\nu}_j$ ), processes of electron capture on protons ( $e^- + p \rightarrow n + \nu_e$ ) and positron capture on neutrons ( $e^+ + n \rightarrow p + \bar{\nu}_e$ ), and nucleon-nucleon bremsstrahlung ( $NN \rightarrow NN + \nu_j + \bar{\nu}_j$ ) for  $j = e, \nu, \tau$ . Also neutrinos of similar energies are naturally expected in the accretion disk during the collapse or merger. As known, properties of neutrinos get modified when they propagate in this magnetized fireball and, depending on the neutrino flavor, would feel a different effective potential because electron neutrino ( $\nu_e$ ) interacts with electrons via both neutral and charged currents (CC), whereas muon ( $\nu_\mu$ ) and tau ( $\nu_\tau$ ) neutrinos interact only via the neutral current (NC). This would induce a coherent effect in which maximal conversion of  $\nu_e$  into  $\nu_\mu$  ( $\nu_\tau$ ) takes place even for a small intrinsic mixing angle (Wolfenstein 1978). Although thermal neutrino oscillations have been widely studied in the literature for different scenarios (Ruffert & Janka 1999; Goodman et al. 1987; Volkas & Wong 2000; Dasgupta et al. 2008; Janka et al. 2012; Sahu and D’Olivo 2005; D’Olivo & Nieves 1996b; Erdas & Isola 2000; D’Olivo & Nieves 1997; D’Olivo et al. 2003; Erdas 2009; Sahu et al. 2009a,b), and even for  $O(m_W^{-4})$  corrections to the neutrino dispersion relation (Nötzold & Raffelt 1988; Enquist et al. 1991; D’Olivo et al. 1992), a full analysis of both the thermal and subphotospheric neutrino oscillations over the dynamics of the fireball GRB as neutrons abundance is comparable to that of protons and the magnetic field is as strong as the

critical magnetic field has not been performed. In this paper, we show that neutrino oscillations have important implications for the evolution of the fireball when it is highly magnetized and has a considerable amount of neutrons and protons. Firstly, we compute the neutrino self-energy by using real time formalism of finite temperature field theory (Nieves 1994; Weldon 1982; D’Olivo et al. 2003; Erdas & Feldman 1990) and Schwinger’s proper-time method (Schwinger 1951). Then, taking into account the strong, moderate and weak magnetic field approximation, we compute, analyze and compare the neutrino effective potential up to order  $m_W^{-4}$  at all the regimes. Next, we derive the resonance conditions to find the range of values for which MeV - GeV neutrinos oscillate resonantly and then we do a full analysis of two- (solar, atmospheric and accelerator parameters) and three-neutrino mixing, even though we estimate the neutrino flavor ratio in the jet and on Earth. After that, we derive the charged-neutrality condition so that together with the resonance condition we can constrain the baryon density as well as the protons and neutrons abundance in the dynamics and evolution of a magnetized fireball. The results are also discussed in terms of magnetization degree.

We hereafter use  $Q_x \equiv Q/10^x$  in cgs units and  $c = \hbar=k=1$  in natural units

## 2. Neutrino Effective Potential

We use the finite temperature field theory formalism to study the effect of heat bath on the propagation of elementary particles (D’Olivo & Nieves 1996a; Tututi et al. 2002). The effect of magnetic field is taken into account through Schwinger’s proper-time method (Schwinger 1951). The effective potential of a particle is calculated from the real part of its self-energy diagram. The neutrino field equation in a magnetized medium is

$$[k - \Sigma(k)]\Psi_L = 0, \quad (1)$$

where the neutrino self-energy operator  $\Sigma(k)$  is a Lorentz scalar which depends on the characterized parameters of the medium, for instance, chemical potential, particle density,

temperature, magnetic field, etc. For our purpose,  $\Sigma(k)$  can be formed by

$$\Sigma(k) = \mathcal{R} \left( a_{\parallel} k_{\parallel} + a_{\perp} k_{\perp} + b \not{u} + c \not{b} \right) \mathcal{L}, \quad (2)$$

where  $k_{\parallel}^{\mu} = (k^0, k^3)$ ,  $k_{\perp}^{\mu} = (k^1, k^2)$  and  $u^{\mu}$  stands for the 4-velocity of the center-of-mass of the medium given by  $u^{\mu} = (1, \mathbf{0})$ . The projection operators are conventionally defined as  $\mathcal{R} = \frac{1}{2}(1 + \gamma_5)$  and  $\mathcal{L} = \frac{1}{2}(1 - \gamma_5)$ . The effect of the magnetic field enters through the 4-vector  $b^{\mu}$  which is given by  $b^{\mu} = (0, \hat{\mathbf{b}})$ . The background classical magnetic field vector is along the  $z$ -axis and consequently  $b^{\mu} = (0, 0, 0, 1)$ . So using the four vectors  $u^{\mu}$  and  $b^{\mu}$  we can express

$$k_{\parallel} = k_0 \not{u} - k_3 \not{b}, \quad (3)$$

and the self-energy can be expressed in terms of three independent four-vectors  $k_{\perp}^{\mu}$ ,  $u^{\mu}$  and  $b^{\mu}$ .

Therefore we can write ( $\Sigma = \mathcal{R} \tilde{\Sigma} \mathcal{L}$ )

$$\tilde{\Sigma} = a_{\perp} k_{\perp} + b \not{u} + c \not{b}. \quad (4)$$

The neutrino self-energy in a magnetic background can be found from Eq. (1)

$$\det[k - \Sigma(k)] = 0. \quad (5)$$

Using the Dirac algebra, the dispersion relation,  $V_{eff} = k_0 - |\mathbf{k}|$ , as a function of Lorentz scalars can be written as

$$V_{eff} = b - c \cos \phi - a_{\perp} |\mathbf{k}| \sin^2 \phi, \quad (6)$$

where  $\phi$  is the angle between the neutrino momentum and the magnetic field vector. Now the Lorentz scalars  $a$ ,  $b$  and  $c$  which are functions of neutrino energy, momentum and magnetic field can be calculated from the neutrino self-energy due to CC and NC interactions of neutrino with the background particles.

## 2.1. One-loop neutrino self-energy

Let us consider one-loop corrections to the neutrino self-energy in the presence of a magnetic field. The one-loop neutrino self-energy comes from three pieces (Bravo and Sahu 2007; Elizalde et al. 2004; Erdas et al. 1998; Sahu et al. 2009a,b), one coming from the  $W$ -exchange diagram which we will call  $\Sigma_W(k)$  (Fig. 1(a)), one from the  $Z$ -exchange diagram which will be denoted by  $\Sigma_Z(k)$  (Fig. 1(b)) and one from the tadpole diagram which we will designate by  $\Sigma_t(k)$  (Fig. 1(c)). The total neutrino self-energy in a magnetized medium then becomes

$$\Sigma(k) = \Sigma_W(k) + \Sigma_Z(k) + \Sigma_t(k). \quad (7)$$

The  $W$ -exchange diagram for the one-loop self-energy is

$$-i\Sigma_W(k) = \mathcal{R} \left[ \int \frac{d^4 p}{(2\pi)^4} \left( \frac{-ig}{\sqrt{2}} \right) \gamma_\mu iS_\ell(p) \left( \frac{-ig}{\sqrt{2}} \right) \gamma_\nu iW^{\mu\nu}(q) \right] \mathcal{L}, \quad (8)$$

where  $g^2 = 4\sqrt{2}G_F m_W^2$  is the weak coupling constant,  $W^{\mu\nu}$  depicts the  $W$ -boson propagator which in the  $eB \ll m_W^2$  limit and in unitary gauge is given by (Erdas et al. 1998; Sahu et al. 2009b)

$$W^{\mu\nu}(q) = \frac{g^{\mu\nu}}{m_W^2} \left( 1 + \frac{q^2}{m_W^2} \right) - \frac{q^\mu q^\nu}{M_W^4} + \frac{3ie}{2m_W^4} F^{\mu\nu}, \quad (9)$$

here  $m_W$  is the  $W$ -boson mass,  $g^{\mu\nu}$  is the metric tensor and  $F^{\mu\nu}$  is the electromagnetic field tensor.  $S_\ell(p)$  stands for the charged lepton propagator which can be separated in to two charged propagators; one in presence of a uniform background magnetic field ( $S_\ell^0(p)$ ) and the other in a magnetized medium ( $S_\ell^\beta(p)$ ) and can be written as

$$S_\ell(p) = S_\ell^0(p) + S_\ell^\beta(p). \quad (10)$$

Assuming that the  $z$ -axis points in the direction of the magnetic field  $B$ , we can express the charged lepton propagator in presence of a uniform background magnetic field as

$$iS_\ell^0(p) = \int_0^\infty e^{\phi(p,s)} G(p,s) ds, \quad (11)$$



where the functions  $\phi(p, s)$  and  $G(p, s)$  are give by

$$\begin{aligned}\phi(p, s) &= is(p_0^2 - m_\ell^2) - is[p_3^2 + \frac{\tan z}{z} p_\perp^2], \\ G(p, s) &= \sec^2 z [\mathbf{A} + i\mathbf{B}\gamma_5 + m_\ell(\cos^2 z - i\Sigma^3 \sin z \cos z)] ,\end{aligned}\quad (12)$$

where  $m_\ell$  is the mass of the charged lepton,  $p_\parallel^2 = p_0^2 - p_3^2$  and  $p_\perp^2 = p_1^2 + p_2^2$  are the projections of the momentum on the magnetic field direction and  $z = eBs$ , being  $e$  the magnitude of the electron charge. Additionally, the covariant vectors are given as follows  $A_\mu = p_\mu - \sin^2 z(p \cdot u u_\mu - p \cdot b b_\mu)$ ,  $B_\mu = \sin z \cos z(p \cdot u b_\mu - p \cdot b u_\mu)$ , and  $\Sigma^3 = \gamma_5 \mathbf{b} \mathbf{u}$ .

On the other hand, the charged lepton propagator in a magnetized medium is given by

$$S_\ell^\beta(p) = i\eta_F(p \cdot u) \int_{-\infty}^{\infty} e^{\phi(p,s)} G(p, s) ds, \quad (13)$$

here  $\eta_F(p \cdot u)$  contains the distribution functions of the particles in the medium which can be written as

$$\eta_F(p \cdot u) = \frac{\theta(p \cdot u)}{e^{\beta(p \cdot u - \mu_\ell)} + 1} + \frac{\theta(-p \cdot u)}{e^{-\beta(p \cdot u - \mu_\ell)} + 1}, \quad (14)$$

where  $\beta$  and  $\mu_\ell$  are the inverse of the medium temperature and the chemical potential of the charged lepton, respectively.

The Z-exchange diagram for the one-loop self-energy is

$$-i\Sigma_Z(k) = \mathcal{R} \left[ \int \frac{d^4 p}{(2\pi)^4} \left( \frac{-ig}{\sqrt{2} \cos \theta_W} \right) \gamma_\mu iS_{\nu_\ell}(p) \left( \frac{-ig}{\sqrt{2} \cos \theta_W} \right) \gamma_\nu iZ^{\mu\nu}(q) \right] \mathcal{L}, \quad (15)$$

where  $\theta_W$  is the Weinberg angle,  $Z^{\mu\nu}(q)$  is the Z-boson propagator in vacuum,  $S_{\nu_\ell}$  is the neutrino propagator in a thermal bath of neutrinos.

The Tadpole diagram for the one-loop self-energy is

$$i\Sigma_t(k) = \mathcal{R} \left[ \left( \frac{g}{2 \cos \theta_W} \right)^2 \gamma_\mu iZ^{\mu\nu}(0) \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [\gamma_\nu (C_V + C_A \gamma_5) iS_\ell(p)] \right] \mathcal{L}, \quad (16)$$

where the quantities  $C_V$  and  $C_A$  are the vector and axial-vector coupling constants which come in the neutral-current interaction of electrons, protons ( $p$ ), neutrons ( $n$ ) and neutrinos with the  $Z$  boson. Their forms are as follows

$$C_V = \begin{cases} -\frac{1}{2} + 2 \sin^2 \theta_W & e \\ \frac{1}{2} & \nu \\ \frac{1}{2} - 2 \sin^2 \theta_W & p \\ -\frac{1}{2} & n \end{cases}, \quad (17)$$

and

$$C_A = \begin{cases} -\frac{1}{2} & \nu, p \\ \frac{1}{2} & e, n \end{cases}. \quad (18)$$

By evaluating eq. (8) explicitly we obtain

$$Re\Sigma_W(k) = \mathcal{R} [a_{W\perp} k_\perp + b_W \not{t} + c_W \not{b}] \mathcal{L}, \quad (19)$$

where the Lorentz scalars are given by

$$\begin{aligned} a_{W\perp} = & -\frac{\sqrt{2}G_F}{m_W^2} \left[ \left\{ E_{\nu_e} (n_e - \bar{n}_e) + k_3 (n_e^0 - \bar{n}_e^0) \right\} \right. \\ & \left. + \frac{eB}{2\pi^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left( \frac{m_e^2}{E_n} - \frac{H}{E_n} \right) (f_{e,n} + \bar{f}_{e,n}) \right], \end{aligned} \quad (20)$$

$$\begin{aligned} b_W = & \sqrt{2}G_F \left[ \left( 1 + \frac{3}{2} \frac{m_e^2}{m_W^2} + \frac{E_{\nu_e}^2}{m_W^2} \right) (n_e - \bar{n}_e) + \left( \frac{eB}{m_W^2} + \frac{E_{\nu_e} k_3}{m_W^2} \right) (n_e^0 - \bar{n}_e^0) \right. \\ & \left. - \frac{eB}{2\pi^2 m_W^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2k_3 E_n \delta_{n,0} + 2E_{\nu_e} \left( E_n - \frac{m_e^2}{2E_n} \right) \right\} (f_{e,n} + \bar{f}_{e,n}) \right] \end{aligned} \quad (21)$$

and

$$\begin{aligned} c_W = & \sqrt{2}G_F \left[ \left( 1 + \frac{1}{2} \frac{m_e^2}{m_W^2} - \frac{k_3^2}{m_W^2} \right) (n_e^0 - \bar{n}_e^0) + \left( \frac{eB}{m_W^2} - \frac{E_{\nu_e} k_3}{m_W^2} \right) (n_e - \bar{n}_e) \right. \\ & \left. - \frac{eB}{2\pi^2 m_W^2} \int_0^\infty dp_3 \sum_{n=0}^\infty (2 - \delta_{n,0}) \left\{ 2E_{\nu_e} \left( E_n - \frac{m_e^2}{2E_n} \right) \delta_{n,0} \right. \right. \end{aligned}$$

$$+2k_3 \left( E_n - \frac{3}{2} \frac{m_e^2}{E_n} - \frac{H}{E_n} \right) \} (f_{e,n} + \bar{f}_{e,n}) \Big]. \quad (22)$$

where the electron number density and electron distribution function are

$$n_e(\mu, T, B) = \frac{eB}{2\pi^2} \sum_{n=0}^{\infty} (2 - \delta_{n,0}) \int_0^{\infty} \frac{dp_3}{e^{\beta(E_{e,n}-\mu)} + 1}, \quad (23)$$

and

$$f(E_{e,n}, \mu) = \frac{1}{e^{\beta(E_{e,n}-\mu)} + 1}, \quad (24)$$

respectively, with  $\bar{f}_{e,n}(\mu, T) = f_{e,n}(-\mu, T)$  and  $E_{e,n} = \sqrt{p_3^2 + m_e^2 + H}$  with  $H = 2neB$ . We can also express the Eq. (15) (Z-exchange) as

$$Re\Sigma_Z(k) = \mathcal{R}(a_Z k + b_Z \mathbf{t}) \mathcal{L}, \quad (25)$$

and explicit evaluation gives (D’Olivo & Nieves 1994)

$$a_Z = \sqrt{2}G_F \left[ \frac{E_{\nu_e}}{m_Z^2} (n_{\nu_e} - \bar{n}_{\nu_e}) + \frac{2}{3} \frac{1}{m_Z^2} \left( \langle E_{\nu_e} \rangle n_{\nu_e} + \langle \bar{E}_{\nu_e} \rangle \bar{n}_{\nu_e} \right) \right], \quad (26)$$

and

$$b_Z = \sqrt{2}G_F \left[ (n_{\nu_e} - \bar{n}_{\nu_e}) - \frac{8E_{\nu}}{3m_Z^2} \left( \langle E_{\nu_e} \rangle n_{\nu_e} + \langle \bar{E}_{\nu_e} \rangle \bar{n}_{\nu_e} \right) \right], \quad (27)$$

where the four vector  $k$  can be decomposed into the four vectors  $\mathbf{t}$  and  $\mathbf{b}$  in accordance with Eq. (3).

From the tadpole diagram, Eq. (16), we obtain

$$\begin{aligned} Re\Sigma_t(k) = & \sqrt{2}G_F \mathcal{R} \left[ \left\{ C_{V_e}(n_e - \bar{n}_e) + C_{V_p}(n_p - \bar{n}_p) + C_{V_n}(n_n - \bar{n}_n) + (n_{\nu_e} - \bar{n}_{\nu_e}) \right. \right. \\ & \left. \left. + (n_{\nu_\mu} - \bar{n}_{\nu_\mu}) + (n_{\nu_\tau} - \bar{n}_{\nu_\tau}) \right\} \mathbf{t} - C_{A_e}(n_e^0 - \bar{n}_e^0) \mathbf{b} \right] \mathcal{L}. \end{aligned} \quad (28)$$

For anti-neutrinos we must change  $(n_x - \bar{n}_x)$  by  $-(n_x - \bar{n}_x)$ . The different contributions to the neutrino self-energy up to order the  $1/m_W^4$  have been calculated in a background of  $\gamma$ ,  $e^\pm$ , free baryons, neutrinos and anti-neutrinos. The effective potential that is applicable to neutrino oscillations in matter is  $V_{eff} = V_e - V_{\mu,\tau}$  which depends only on electron density (Wolfenstein 1978; D’Olivo et

al. 1992), assuming that neutrinos propagate in the same direction of the magnetic field ( $\phi = 0$ ) and with  $k_3 = E_{\nu_e}$ , we derive the neutrino effective potential in all the regimes of the magnetic field: strong  $\Omega_B = eB/m_e^2 \gg 1$ , moderate  $\Omega_B = eB/m_e^2 > 1$  (above  $B_c$ ) and  $\Omega_B = eB/m_e^2 \leq 1$  (below  $B_c$ ), and weak  $\Omega_B = eB/m_e^2 \ll 1$  regime, where  $B_c = m_e^2/e = 4.414 \times 10^{13}$  G is the critical magnetic field. In the following subsections we will show the neutrino effective potential in all the regimes (strong, moderate and weak magnetic field limit)(Erdaş 2009); the calculations are explicitly shown in the appendix. Using the typical fireball values at the initial stage and the phase of acceleration; temperature in the range 1- 10 MeV at initial radius  $r_0 \simeq 10^{6.5} - 10^{7.5}$  cm for thermal MeV neutrinos (Beloborodov 2003a; Ruffert & Janka 1999; Koers & Wijers 2005; Janka et al. 2012) and in the range 0.1 - 1 MeV at radius  $r \simeq 10^{8.5} - 10^{9.5}$  cm for quasi-thermal GeV neutrinos (Bahcall & Meszaros 2000; Meszaros & Rees 2000; Razzaque & Meszaros 2006a,b; Murase et al. 2013), we plot the effective potential in each limit as shown in figs. 2, 3, 4 and 5. Although we are going to do a full analysis of neutrino oscillations in a fireball endowed with moderate magnetic fields which is more favored for GRB central engines (Beloborodov 2003a; Mészáros 2012), a brief and additional description at the strong and weak field limit as well as a comparison of effective potentials at all the regime will be given as follows.

## 2.2. Strong Magnetic field: $\Omega_B \gg 1$

In the strong magnetic field approximation ( $m_e^2 \ll T^2 \ll \Omega_B m_e^2$ ), leptons are all confined to the lowest Landau level ( $n=0$ ), then only this level will contribute to the potential and the energy of these leptons will be independent of the magnetic field. The neutrino effective potential at the strong magnetic field is given by

$$V_{eff,S} = A_e \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l K_1(\sigma_l) \left\{ \frac{m_e^2}{m_W^2} \left( 1 + 4 \frac{E_{\nu}^2}{m_e^2} \right) \right\} - 3 \frac{m_e^2}{m_W^2} \frac{E_{\nu}}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l K_0(\sigma_l) \right]. \quad (29)$$

In fig. 2, we plot the effective potential at the strong field limit (eq. 29) as a function of temperature (left-hand figure above), magnetic field (right-hand figure above) and chemical potential (figure below). For these plots we take into account the neutrino energy 10 MeV and the values of temperature, magnetic field and chemical potential in the range 1 to 10 MeV,  $10^3$  to  $10^5 B_c$  and  $10^{-4}$  eV to 4.5 keV, respectively. As shown, the effective potential is a quasi-constant function of temperature, and an increasing function of magnetic field and chemical potential

### 2.3. Moderate Magnetic field: $\Omega_B > 1$ and $\Omega_B \leq 1$

In the moderate field approximation ( $m_e^2 < \Omega_B m_e^2 \leq T^2$ ), leptons start to occupy the next Landau levels ( $n=1, 2, 3 \dots$ ) which have a separation that is directly proportional to magnetic field. For this case each of these levels will contribute to the effective potential and the energy of leptons are directly proportional to magnetic field. The neutrino effective potential in the moderate magnetic field is written as

$$V_{eff} = A_e \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ \frac{m_e^2}{m_W^2} \left( 1 + 4 \frac{E_\nu^2}{m_e^2} \right) K_1(\sigma_l) + \sum_{n=1}^{\infty} \lambda_n \left( 2 + \frac{m_e^2}{m_W^2} \left( 3 - 2\Omega_B + 4 \frac{E_\nu^2}{m_e^2} \right) \right) K_1(\sigma_l \lambda_n) \right\} \right. \\ \left. - 4 \frac{m_e^2}{m_W^2} \frac{E_\nu}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ \frac{3}{4} K_0(\sigma_l) + \sum_{n=1}^{\infty} \lambda_n^2 K_0(\sigma_l \lambda_n) \right\} \right],$$

with

$$\lambda^2 = \begin{cases} 2 n \Omega_B & \text{for } \Omega_B > 1 \\ 1 + 2 n \Omega_B & \text{for } \Omega_B \leq 1: \end{cases} \quad (30)$$

In fig. 3, we have plotted the neutrino effective potential at the moderate field limit above  $B_c$  (eq. 30) as a function of temperature (left-hand figure above), magnetic field (right-hand figure above) and chemical potential (figure below). For these plots we take into account the neutrino energy 10 MeV and the values of temperature, magnetic field and chemical potential in the range 1 to 10 MeV,  $B_c < B \leq 10^2 B_c$  and  $10^{-4}$  eV to 4.5 keV, respectively. As shown, the effective potential as a function of temperature has two different behaviors. Firstly, it is an increasing function of temperature in the range 1 to 3 MeV for values of magnetic field  $5B_c$  and  $10B_c$ , and in the range

1 to 7 MeV for  $50B_c$  and  $100B_c$  and secondly, it tends to be constant for values of temperature greater than 3 MeV for  $B = 5B_c$  and  $B = 10B_c$ , and 7 MeV for  $B = 50B_c$  and  $B = 100B_c$ . In this figure it is also seen that the effective potential is an increasing function for both the magnetic field and chemical potential.

In fig. 4, we have plotted the neutrino effective potential at the moderate field limit below  $B_c$  (eq. 30) as a function of temperature (figure above), magnetic field (middle figure) and chemical potential (figure below) for neutrino energies  $E_\nu = 10$  (left column figures) MeV and  $E_\nu = 10$  GeV (right column figures). For these plots we take into account the values of the magnetic field and chemical potential in the range  $10^{-3}B_c < B \leq B_c$  and  $10^{-4}$  eV to 4.5 keV, respectively, and two ranges of temperatures;  $0.1 \text{ MeV} \leq T \leq 1 \text{ MeV}$  (right column figures) and  $1 \text{ MeV} \leq T \leq 10 \text{ MeV}$  (left column figures). In the top figures, the behavior of the effective potential as a function of temperature has a multifunctional dependence which depends on the values of the magnetic field. For instance, in the right-hand figure, for  $B=0.5 B_c$  it is a dramatically increasing function, for  $B=10^{-2} B_c$  it is just a steadily increasing function and for the smaller values of  $B$ , it becomes a decreasing function and in the left-hand figure, for  $B=0.5 B_c$  it is a steadily increasing function and as  $B$  decreases the effective potential gradually becomes a decreasing function. In the middle figures, the effective potential is a dramatically (left-hand figure) and (right-hand figure) steadily increasing function of the magnetic field regardless of the values of temperature. In the bottom figures, the effective potential represented by means of an increasing function of the chemical potential shows the same behavior but at different energy ranges.

#### 2.4. Weak Magnetic field: $\Omega_B \ll 1$

In the weak field approximation ( $\Omega_B m_e^2 \ll m_e^2$ ), all Landau levels are occupied and overlapped each other, therefore a good description of these levels is to take the approximation

$\sum_n \rightarrow \int dn$ . For this case, the neutrino effective potential is

$$V_{eff,W} = A_e \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ \left( 2 + \frac{m_e^2}{m_W^2} \left( 3 + 4 \frac{E_\nu^2}{m_e^2} \right) \right) \left( \frac{K_0(\sigma_l)}{\sigma_l} + 2 \frac{K_1(\sigma_l)}{\sigma_l^2} \right) \Omega_B^{-1} - 2 \left( 1 + \frac{m_e^2}{m_W^2} \right) K_1(\sigma_l) \right\} \right. \\ \left. - 4 \frac{m_e^2}{m_W^2} \frac{E_\nu}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ \left( \frac{2}{\sigma_l^2 \Omega_B} - \frac{1}{4} \right) K_0(\sigma_l) + \left( 1 + \frac{4}{\sigma_l^2} \right) \frac{K_1(\sigma_l)}{\sigma_l} \Omega_B^{-1} \right\} \right]. \quad (31)$$

As shown in fig. 5 the neutrino effective potential at the weak field limit (eq. 31) as a function of temperature (figures above) and chemical potential (figures below) for neutrino energies  $E_\nu=10$  MeV (left-hand figures) and  $E_\nu=10$  GeV (left-hand figures) is plotted. Due to the strength of the magnetic field it is quite small and any variation will produce insignificant changes in the effective potential; we only show the magnetic field contribution, i.e. by subtracting the effective potential with  $B=0$ , which shows that the potential is a decreasing function of temperature and an increasing function of chemical potential, and that the magnetic contribution is the opposite as compared with the medium contribution. The effective potential in this regime differs from that calculated by Sahu et al. (2009b), because the authors took the solution of dispersion relation  $k_3 = -E_{\nu_e}$  instead of  $k_3 = E_{\nu_e}$ .

## 2.5. Comparison of effective potentials at all the regimes

We plot the neutrino effective potentials as a function of temperature (left-hand figure) and chemical potential (right-hand figure) in a strong, moderate and weak magnetic field approximation (fig. 6) which are given in eqs. (29), (30) and (31), respectively. As shown in fig. 6, the effective potential lies between  $\sim 10^{-11}$  and  $\sim 10^{-8}$  eV for a temperature in the range 1 to 8 MeV. One can see from this plot several features. i) at  $T \sim 1$  MeV, the effective potentials at moderate (above and below  $B_c$ ) field limit are quite close at  $\sim 5 \times 10^{-10}$  eV, however as temperature increases the separation between them also increases and for strong and weak field limit, the distance between the effective potentials is more than two orders of magnitude. ii) As

temperature increases the effective potentials at the strong and moderate (above  $B_c$ ) field limits become closer to each other. Dividing this plot in two regions,  $T < 3$  MeV and  $T \geq 3$  MeV, we can argue that for the given values of the magnetic field and temperature, in the first region  $m_e^2 \Omega_B \simeq T^2$ , even though being slightly lower at  $T \sim 1.4$  MeV and as temperature becomes larger than the magnetic field,  $m_e^2 \Omega_B \ll T^2$ , a difference between both effective potentials for each value of temperature is observed. This difference, which is of the same order of magnitude, comes from the contribution of excited Landau levels ( $n=1,2 \dots$ ). iii) The effective potential at the weak field limit is a dramatically increasing function of temperature which tends at the moderate limit below  $B_c$ . iv) The effective potential at strong field limit decreases very gradually (quasi constantly) as temperature increases, in fact for the range of temperature considered, the effective potential is almost invariant to any thermal contribution. Also it is an increasing function of the chemical potential in the range  $0.1 - 4 \times 10^3$  eV for all the regimes of the magnetic fields. By comparison the effective potential at strong (maximum value) and weak (minimum value) field approximation, one can see that the last approximation is smaller by three orders of magnitude than the first one. Additionally, we plot the contribution of  $m_W^{-4}$  terms to the neutrino effective potential as shown in fig. 7. As can be seen in eqs. (29), (30) and (31), there are two common terms which depend on  $m_W^{-4}$  and that contribute to the effective potentials ;  $(-1)^l \cosh(\alpha) K_0(\sigma)$  and  $4 E_\nu^2 / m_W^2$ . In the first case, we compare the terms  $(-1)^l \sinh(\alpha) K_1(\sigma)$  and  $(-1)^l \cosh(\alpha) K_0(\sigma)$  (left-hand figure) where the first term comes from particle-antiparticle asymmetry ( $n_e - \bar{n}_e$ ) and the second one from ( $n_e + \bar{n}_e$ ). From this figure, one can observe that the  $(n_e - \bar{n}_e) / (n_e + \bar{n}_e)$  ratio is directly proportional to chemical potential and for  $\mu \leq 10^2$  eV the  $m_W^{-4}$  term begins to be dominant achieving a minimum value of 7 orders of magnitude for  $\mu = 10^{-3}$  eV. In other words, as  $\mu$  decreases the correction of order  $m_W^{-4}$  is important and dominant over  $m_W^{-2}$  term, even though one can deduce that for  $\mu = 0$ ,  $n_e - \bar{n}_e = 0$ , then the only contribution would come from the term  $O(m_W^{-4})$ . In the second case we plot  $4 E_\nu^2 / m_W^2$  as a function of  $E_\nu$  (right-hand figure). From this plot, you can notice that this term in comparison with unity starts to contribute for neutrinos with



energies of some tens of GeV. As shown above, these two terms with the values of temperature, chemical potential and neutrino energy are relevant to the neutrino effective potential. From the previous analysis can be observed that regardless of the magnetic field limit or relativistic temperature or chemical potential, the neutrino effective potential is positive, therefore due to its positivity ( $V_{eff,k} > 0$  for  $k=S, M$  and  $W$ ), MeV - GeV neutrinos can oscillate resonantly.

### 3. Neutrino Mixing and Resonance Condition

In the following subsections we are going to consider the best fit values of the neutrino oscillation parameters for two-neutrino mixing (solar, atmospheric and accelerator) and three-neutrino mixing.

#### 3.1. Two-Neutrino Mixing

Here we consider the neutrino oscillation process  $\nu_e \leftrightarrow \nu_{\mu,\tau}$ . The evolution equation for the propagation of neutrinos in the medium is given by (Fraija 2014a)

$$i \begin{pmatrix} \dot{\nu}_e \\ \dot{\nu}_\mu \end{pmatrix} = \begin{pmatrix} V_{eff} - \Delta \cos 2\theta & \frac{\Delta}{2} \sin 2\theta \\ \frac{\Delta}{2} \sin 2\theta & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad (32)$$

where  $\Delta = \delta m^2 / 2E_\nu$ ,  $\delta m^2$  is the mass difference,  $V_{eff}$  is the neutrino effective potential between  $V_{\nu_e}$  and  $V_{\nu_{\mu,\tau}}$  (eq. 30),  $E_\nu$  is the neutrino energy and  $\theta$  is the neutrino mixing angle. The conversion probability for the above process at a time  $t$  is given by

$$P_{\nu_e \rightarrow \nu_\mu(\nu_\tau)}(t) = \frac{\Delta^2 \sin^2 2\theta}{\omega^2} \sin^2 \left( \frac{\omega t}{2} \right), \quad (33)$$

with

$$\omega = \sqrt{(V_{eff} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}. \quad (34)$$

The oscillation length for the neutrino is given by

$$l_{osc} = \frac{l_v}{\sqrt{\cos^2 2\theta \left(1 - \frac{V_{eff}}{\Delta \cos 2\theta}\right)^2 + \sin^2 2\theta}}, \quad (35)$$

where  $l_v = 2\pi/\Delta$  is the vacuum oscillation length. Applying the resonance condition given by

$$\begin{aligned} V_{eff} &= \Delta \cos 2\theta \\ &= 5 \times 10^{-7} \text{ eV} \frac{\delta m_{eV}^2}{E_{\nu, MeV}} \cos 2\theta, \end{aligned} \quad (36)$$

we obtain that the resonance length ( $l_{res}$ ) can be written as

$$l_{res} = \frac{l_v}{\sin 2\theta}. \quad (37)$$

In eq. (36) the neutrino effective potential depends on the chemical potential, temperature, the neutrino energy and the oscillation parameters (mass differences and the mixing angles). We will use the following parameters for this analysis:

**Solar Neutrinos:** A two-flavor neutrino oscillation analysis yielded  $\delta m^2 = (5.6_{-1.4}^{+1.9}) \times 10^{-5} \text{ eV}^2$  and  $\tan^2 \theta = 0.427_{-0.029}^{+0.033}$  (Aharmin et al. 2011).

**Atmospheric Neutrinos:** Under a two-flavor disappearance model with separate mixing parameters between neutrinos and antineutrinos the following parameters for the SK-I + II + III data  $\delta m^2 = (2.1_{-0.4}^{+0.9}) \times 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta = 1.0_{-0.07}^{+0.00}$  were found. (Abe et al. 2011a).

**Accelerator Parameters (Short baselines):** Church et al. (2002) found two well defined regions of oscillation parameters with either  $\delta m^2 \approx 7 \text{ eV}^2$  or  $\delta m^2 < 1 \text{ eV}^2$  compatible with both LAND and KARMEN experiments, for the complementary confidence. In addition, MiniBooNE found evidence of oscillations in the 0.1 to 1.0  $\text{eV}^2$ , which are consistent with LSND results (Athanasopoulos et al. 1996, 1998).

### 3.2. Three-Neutrino Mixing

To determine the neutrino oscillation probabilities we have to solve the evolution equation of the neutrino system in the matter. In a three-flavor framework, this equation is given by

$$i \frac{d\vec{\nu}}{dt} = H \vec{\nu}, \quad (38)$$

and the state vector in the flavor basis is defined as

$$\vec{\nu} \equiv (\nu_e, \nu_\mu, \nu_\tau)^T. \quad (39)$$

The effective Hamiltonian is

$$H = U \cdot H_0^d \cdot U^\dagger + \text{diag}(V_{eff}, 0, 0), \quad (40)$$

with

$$H_0^d = \frac{1}{2E_\nu} \text{diag}(-\delta m_{21}^2, 0, \delta m_{32}^2). \quad (41)$$

Here  $V_{eff}$  is the effective potential (eq. 30) and  $U$  is the three neutrino mixing matrix given by (Gonzalez-Garcia 2003; Akhmedov et al. 2004; Gonzalez-Garcia & Maltoni 2008; Gonzalez-Garcia 2011),

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix}, \quad (42)$$

where  $s_{ij} = \sin \theta_{ij}$  and  $c_{ij} = \cos \theta_{ij}$ . For anti-neutrinos one has to replace  $U$  by  $U^*$ . The different neutrino probabilities are given as

$$\begin{aligned} P_{ee} &= 1 - 4s_{13,m}^2 c_{13,m}^2 S_{31}, \\ P_{\mu\mu} &= 1 - 4s_{13,m}^2 c_{13,m}^2 s_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32}, \\ P_{\tau\tau} &= 1 - 4s_{13,m}^2 c_{13,m}^2 c_{23}^4 S_{31} - 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} - 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32}, \\ P_{e\mu} &= 4s_{13,m}^2 c_{13,m}^2 s_{23}^2 S_{31}, \end{aligned}$$

$$\begin{aligned}
P_{e\tau} &= 4s_{13,m}^2 c_{13,m}^2 c_{23}^2 S_{31} \\
P_{\mu\tau} &= -4s_{13,m}^2 c_{13,m}^2 s_{23}^2 c_{23}^2 S_{31} + 4s_{13,m}^2 s_{23}^2 c_{23}^2 S_{21} + 4c_{13,m}^2 s_{23}^2 c_{23}^2 S_{32},
\end{aligned} \tag{43}$$

where

$$\sin 2\theta_{13,m} = \frac{\sin 2\theta_{13}}{\sqrt{(\cos 2\theta_{13} - 2E_\nu V_{eff}/\delta m_{32}^2)^2 + (\sin 2\theta_{13})^2}}, \tag{44}$$

and

$$S_{ij} = \sin^2 \left( \frac{\Delta\mu_{ij}^2}{4E_\nu} l_{osc} \right) \tag{45}$$

with

$$\begin{aligned}
\Delta\mu_{21}^2 &= \frac{\delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} - 1 \right) - E_\nu V_{eff} \\
\Delta\mu_{32}^2 &= \frac{\delta m_{32}^2}{2} \left( \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}} + 1 \right) + E_\nu V_{eff} \\
\Delta\mu_{31}^2 &= \delta m_{32}^2 \frac{\sin 2\theta_{13}}{\sin 2\theta_{13,m}},
\end{aligned} \tag{46}$$

and

$$\begin{aligned}
\sin^2 \theta_{13,m} &= \frac{1}{2} \left( 1 - \sqrt{1 - \sin^2 2\theta_{13,m}} \right) \\
\cos^2 \theta_{13,m} &= \frac{1}{2} \left( 1 + \sqrt{1 - \sin^2 2\theta_{13,m}} \right).
\end{aligned} \tag{47}$$

The oscillation length for the neutrino is given by

$$l_{osc} = \frac{l_v}{\sqrt{\cos^2 2\theta_{13} \left( 1 - \frac{2E_\nu V_e}{\delta m_{32}^2 \cos 2\theta_{13}} \right)^2 + \sin^2 2\theta_{13}}}, \tag{48}$$

where  $l_v = 4\pi E_\nu / \delta m_{32}^2$  is the vacuum oscillation length. The resonance condition and resonance length are,

$$V_{eff} - 5 \times 10^{-7} \frac{\delta m_{32,eV}^2}{E_{\nu, MeV}} \cos 2\theta_{13} = 0 \tag{49}$$

and

$$l_{res} = \frac{l_v}{\sin 2\theta_{13}}. \tag{50}$$

Considering the adiabatic condition at the resonance, we can express it as,

$$\begin{aligned}\kappa_{res} &\equiv \frac{2}{\pi} \left( \frac{\delta m_{32}^2}{2E_\nu} \sin 2\theta_{13} \right)^2 \left( \frac{dV_{eff}}{dr} \right)^{-1} \geq 1 \\ &= 3.62 \times 10^{-2} \text{cm}^{-1} \left( \frac{\delta m_{32,eV}^2}{E_{\nu,MeV}} \sin 2\theta_{13} \right)^2 \frac{1}{V'} \geq 1,\end{aligned}\quad (51)$$

where

$$V' = \Omega_B \left[ \frac{dV_{eff,1}}{dr} - 3.16 \times 10^{-10} E_{\nu,MeV} \frac{dV_{eff,2}}{dr} \right]. \quad (52)$$

and

$$\begin{aligned}V_{eff,1} &= \sum_{l=0}^{\infty} (-1)^l \sin \alpha_l \left\{ \frac{m_e^2}{m_W^2} \left( 1 + 4 \frac{E_\nu^2}{m_e^2} \right) K_1(\sigma_l) \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \lambda_n \left[ 2 + \frac{m_e^2}{m_W^2} \left( 3 + 4 \frac{E_\nu^2}{m_e^2} - 2\Omega_B \right) \right] K_1(\sigma_l \lambda_n) \right\} \\ V_{eff,2} &= \sum_{l=0}^{\infty} (-1)^l \cos \alpha_l \left( \frac{3}{4} K_0(\sigma_l) + \sum_{n=1}^{\infty} \lambda_n^2 K_0(\sigma_l \lambda_n) \right)\end{aligned}\quad (53)$$

We will use the following parameters (Aharmin et al. 2011; Wendell et al. 2010) for this analysis:

$$\begin{aligned}\text{for } \sin_{13}^2 < 0.053 : \delta m_{21}^2 &= (7.41_{-0.19}^{+0.21}) \times 10^{-5} \text{ eV}^2 \text{ and } \tan^2 \theta_{12} = 0.446_{-0.029}^{+0.030} \\ \text{for } \sin_{13}^2 < 0.04 : \delta m_{23}^2 &= (2.1_{-0.2}^{+0.5}) \times 10^{-3} \text{ eV}^2 \text{ and } \sin^2 \theta_{23} = 0.50_{-0.093}^{+0.083}\end{aligned}\quad (54)$$

For a complete description of resonant neutrino oscillations at early fireball stage, we use the values of fireball observables during the initial stage and phase of acceleration. At the initial stage, we consider a fireball endowed with magnetic field  $B = 10 B_c$ , temperature in the range 1 to 5 MeV and thermal neutrino energies  $E_\nu = 1, 5, 20$  and 30 MeV whereas at the acceleration phase the corresponding values considered are: magnetic field  $B = 10^{-4.3} B_c$ , temperature in the range 50 to 500 keV and quasi-thermal GeV neutrino energies  $E_\nu = 1, 10, 20$  and 50 GeV. In both cases we take the best fit of parameters for two- (solar:  $\delta m^2 = 5.6 \times 10^{-5} \text{ eV}^2$  and  $\tan^2 \theta = 0.427$  (Aharmin et al. 2011), atmospheric:  $\delta m^2 = 2.1 \times 10^{-3} \text{ eV}^2$  and  $\sin^2 2\theta = 1.0$  (Abe et al. 2011a), and accelerator:  $\delta m^2 \sim 0.6 \text{ eV}^2$  and  $\sin^2 2\theta = 51.2 \times 10^{-3}$  (Church et al. 2002)) and three ( $\delta m_{32}^2 = 10^{-2.58}$  and  $\theta_{13} = 11^\circ$ ) -neutrino mixing in order to analyze the resonance conditions in

both phases.

From the resonance conditions (eqs. 36 and 49), we have obtained the contour plots of temperature and chemical potential at the initial stage (fig. 8) and phase of acceleration (fig. 9). At the initial phase, chemical potential lies in the range  $7.4 \times 10^{-3}$  to  $3.01$  eV for solar parameters,  $1.01 \times 10^{-2}$  to  $2.84$  eV for atmospheric parameters,  $0.12$  to  $3.1 \times 10^2$  keV for accelerator parameters and  $0.8$  to  $50 \times 10^2$  eV for three-neutrino mixing. One can see that temperature is a decreasing function of chemical potential which gradually increases as neutrino energy is decreased. In addition, we have computed the resonance lengths which are shown in table 1. As shown in this table, the resonance lengths lie in the range  $l_{res} \sim (10^4 \text{ to } 10^8)$  cm which are comparable with the length scale of a fireball. Therefore, depending on the oscillation parameters neutrinos could oscillate resonantly before leaving the fireball. For instance, considering an initial radius of  $10$  km, only neutrinos with low energy will oscillate resonantly for atmospheric, accelerator and three-neutrino mixing but no solar parameters. If we assume an initial radius of  $100$  km, and consider atmospheric and accelerator oscillation parameters, then neutrinos would oscillate resonantly regardless their energies. Once again, assuming a radius of  $1000$  km, thermal neutrinos will oscillate resonantly. In the phase of acceleration, chemical potential lies in the range  $2.2 \times 10^{-2}$  to  $1$  eV for solar,  $4.01 \times 10^{-2}$  to  $1.3$  eV for atmospheric,  $1.1 \times 10^{-2}$  to  $0.5$  keV for accelerator and  $0.5$  to  $6.1$  eV for three-neutrino mixing. One can see that temperature is a decreasing function of chemical potential which increases as neutrino energy increases (decreases) for solar and atmospheric (accelerator) oscillation parameters and it is doubly degenerate for three-neutrino mixing. Further, we have computed the resonance lengths of GeV neutrinos. As shown in table 2, the resonance lengths match the length scale where they were created for three-neutrino mixing. In other words, taking into account the former parameters, multi GeV neutrinos created at  $\sim (10^{11} - 10^{13})$  cm will have simultaneously resonant oscillations. Finally, we have studied the survival and conversion probability for the active-active ( $\nu_{e,\mu,\tau} \leftrightarrow \nu_{e,\mu,\tau}$ ) neutrino oscillations and the three-neutrino mixing. In fig. 10 the survival probability of electron  $P_{ee}$ , muon  $P_{\mu\mu}$  and tau  $P_{\tau\tau}$  neutrino and

Energy (MeV)	$l_{res}$ (cm)			
	Solar	Atmosph.	Accelerat.	Three flavors
1	$4.8 \times 10^6$	$1.2 \times 10^5$	$7.1 \times 10^3$	$2.5 \times 10^5$
5	$2.4 \times 10^7$	$5.9 \times 10^5$	$3.6 \times 10^4$	$1.3 \times 10^6$
20	$1.0 \times 10^8$	$2.4 \times 10^6$	$1.4 \times 10^5$	$5.0 \times 10^7$
30	$1.5 \times 10^8$	$3.5 \times 10^6$	$2.1 \times 10^5$	$7.6 \times 10^7$

Table 1: Resonance lengths of thermal neutrinos for the best fit parameters of two- and three-neutrino mixing.

Energy (GeV)	$l_{res}$ (cm)			
	Solar	Atmosph.	Accelerat.	Three flavors
5	$2.4 \times 10^{10}$	$5.9 \times 10^8$	$3.6 \times 10^7$	$1.3 \times 10^9$
10	$4.8 \times 10^{10}$	$1.2 \times 10^9$	$7.1 \times 10^7$	$2.5 \times 10^9$
20	$9.6 \times 10^{10}$	$2.4 \times 10^9$	$1.4 \times 10^8$	$5.0 \times 10^9$
50	$2.4 \times 10^{11}$	$5.9 \times 10^9$	$3.6 \times 10^8$	$1.3 \times 10^{10}$

Table 2: Resonance lengths of quasi-thermal neutrinos for the best fit parameters of the two- and three-neutrino mixing.

conversion probabilities  $P_{e\mu}$ ,  $P_{e\tau}$  and  $P_{\mu\tau}$  as a function of energy for fixed values of length scale, temperature, magnetic field and neutrino energy is plotted. In the left-hand figures, we use once again the fireball values in the initial stage:  $T=5$  MeV,  $r_0=100$  km (above) and  $r_0=10$  km (below),  $B=10 B_c$ , neutrino energy range 1 to 30 MeV and in the right-hand figures we take into account the values already mentioned for the phase of acceleration:  $T=100$  keV,  $r=10^{11}$  cm (above) and  $r=10^{12}$  cm (below),  $B=10^{-4.3} B_c$ , neutrino energy range 1- 30 GeV. Our analysis shows that  $P_{ee} = 1$ ,  $P_{e\mu} = P_{e\tau} = 0$ , indicating that the propagating electron neutrinos do not oscillate to any other flavor independently of their energies and radii. Instead, it tells us that muon and tau neutrinos oscillate among themselves with equal probability and that the oscillation depends on their energy and distances. As can be seen, the probabilities satisfy the condition

$$\sum_{i=e,\mu,\tau} P_{ei}(\delta m_{32}^2, L) = 1, \quad \sum_{i=e,\mu,\tau} P_{\mu i}(\delta m_{32}^2, L) = 1, \quad \sum_{i=e,\mu,\tau} P_{\tau i}(\delta m_{32}^2, L) = 1. \quad (55)$$

### 3.3. Neutrino Oscillation from Source to Earth

Between the surface of the star and the Earth the flavor ratio  $\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$  is affected by the full three description flavor mixing, which is calculated as follows. The probability for a neutrino to oscillate from a flavor state  $\alpha$  to a flavor state  $\beta$  in a time starting from the emission of neutrino at star  $t=0$ , is given as

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta} &= |\langle \nu_\beta(t) | \nu_\alpha(t=0) \rangle| \\ &= \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\delta m_{ij}^2 l_{osc}}{4 E_\nu} \right). \end{aligned} \quad (56)$$

Using the set of parameters given in eq. (54), we can write the mixing matrix

$$U = \begin{pmatrix} 0.816669 & 0.544650 & 0.190809 \\ -0.504583 & 0.513419 & 0.694115 \\ 0.280085 & -0.663141 & 0.694115 \end{pmatrix}. \quad (57)$$

Additionally, averaging the sin term in the probability to  $\sim 0.5$  for larger distances  $l_{osc}$  (Learned & Pakvasa 1995), the probability matrix for a neutrino flavor vector of  $(\nu_e, \nu_\mu, \nu_\tau)_{source}$  changing



to a flavor vector  $(\nu_e, \nu_\mu, \nu_\tau)_{Earth}$  is given as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{Earth} = \begin{pmatrix} 0.534143 & 0.265544 & 0.200313 \\ 0.265544 & 0.366436 & 0.368020 \\ 0.200313 & 0.368020 & 0.431667 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{source}, \quad (58)$$

for distances longer than the solar system.

## 4. Dynamics of the Fireball

Although we have already introduced general concepts of the fireball model and its dynamics, and also calculated the range of values of chemical potentials and temperatures for which the resonance condition is satisfied, here we are going to quantify or/and estimate the observable quantities in the evolution of the fireball requiring the charged-neutrality condition in addition to resonance condition. In this manner, we are going to constrain the dynamics of the fireball by means of neutrino oscillations.

### 4.1. Initial stage

The initial state of the fireball is magnetized, hot and dense of baryons and pairs  $e^\pm$  in perfect thermodynamic equilibrium with a comoving temperature

$$T' = \left( \frac{L_{52}}{4\pi r_{0,6}^2 a} \right)^{1/4} \simeq 3.8 \text{ MeV}, \quad (59)$$

where  $a = \pi^2 k^4/15 = 7.6 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$  is the radiation density constant,  $L$  is the total isotropic-equivalent energy outflow in the jet and  $r_0$  is the initial radius. Magnetic fields could be estimated by means of the equipartition of the total energy density ( $U$ )

$$B \simeq \sqrt{8\pi\epsilon_B U}, \quad (60)$$

which becomes important as the energy density is dominated by baryons (Beloborodov 2003a)

$$U \simeq \frac{3}{2} \frac{T\rho}{m_p}, \quad (61)$$

and the electrons are degenerated with the charged-neutrality condition,  $n_e(\mu, T, B) - \bar{n}_e(\mu, T, B) = n_p$  where  $n_e(\mu, T, B)$  and  $\bar{n}_e(\mu, T, B)$  are the number densities of electrons and positrons generated in the plasma a temperature  $T$  and magnetic field  $B$ , and  $n_p$  is the number density of protons. Taking into account that the number density of neutrons is comparable to that of protons and from the number density of electrons and positrons at the moderate magnetic field and relativistic temperatures found in section II (eq. B4), the charged-neutrality condition is given by

$$\frac{m_e^3}{\pi^2} \Omega_B \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ K_1(\sigma_l) + 2 \sum_{n=1}^{\infty} \lambda_n K_1(\sigma_l \lambda_n) \right\} = Y_e \frac{\rho}{m_p}, \quad (62)$$

here  $\sigma_l$  and  $\alpha_l$  correspond to the values found for which resonance condition is satisfied,  $\rho$  is the baryon density and  $Y_e = n_p/(n_n + n_p)$  is the proton-to-nucleon ratio. The baryon density is defined by means of the total mass outflow rate in baryon loaded jet  $\dot{M} = 4\pi r_0^2 \rho$  and the dimensionless entropy or baryon load parameter  $\eta = L/\dot{M}$ . The entropy per baryon is conserved in the flow and related to  $\eta$  and  $T$  as  $s/k_B = 4\eta m_p/3T$ . Initially, due to high temperatures  $T \gg m_e$  and charged current reactions

$$e^- + p \rightarrow n + \nu, \quad e^+ + n \rightarrow p + \bar{\nu}, \quad (63)$$

protons convert into neutrons and neutrons into protons, achieving a balance

$$Y_e = \frac{1}{2} + \frac{7\pi^4}{1350 \zeta(5)} \left( \frac{Q/2 - \mu}{T} \right) \quad (64)$$

between the rates of  $e^-$  and  $e^+$  capture by  $n_p$  and  $n_n$  with  $Q$  defined through neutron ( $m_n$ ) and proton ( $m_p$ ) mass,  $Q = m_n - m_p$  (Beloborodov 2003a). Hence, the charged-neutrality condition can be written as

$$\sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ K_1(\sigma_l) + 2 \sum_{n=1}^{\infty} \lambda_n K_1(\sigma_l \lambda_n) \right\} = \frac{\rho \Omega_B^{-1}}{m_p m_e^3} \left[ \frac{\pi^2}{2} + \frac{7\pi^6}{1350 \zeta(5)} \left( \frac{Q/2 - \mu}{T} \right) \right]. \quad (65)$$

It is very important to highlight that the proton-to-nucleon ratio is a function of temperature and chemical potential of the initial stage of the fireball, when the initial optical depth is extremely high and the phase of acceleration has not yet begun.

Taking into account the range of values of chemical potentials and temperatures found from resonance conditions (figs. 8 and 9), considering magnetic fields at the moderate field limit,  $50 B_c$  (left-hand figures) and  $0.1 B_c$  (right-hand figures) and requiring the charged-neutrality condition (eqs. 62 and 65), we plot contour lines of baryon density ( $\rho$ ) (above) and the proton-to-nucleon ratio ( $Y_e$ ) (below) as a function of temperature as shown in fig. 11. From the figures above, one can see that the highest line of baryon density increases gradually achieving a maximum density of  $7.2 \times 10^7 \text{ g cm}^{-3}$  (left-hand figure) and  $5.6 \times 10^6 \text{ g cm}^{-3}$  (right-hand figure) for  $2 \text{ MeV} \leq T \leq 3 \text{ MeV}$  and  $\mu = 160 \text{ keV}$ . This value of chemical potential is the largest one obtained from the resonance condition, hence resonant oscillations are suppressed for densities larger than these. From the figures below one can see that proton-to-nucleon ratio is a decreasing function of temperature achieving a minimum value of  $\sim 0.52$  and  $0.53$  for baryon density  $\simeq 10^8 \text{ g cm}^{-3}$  (left-hand figure) and  $\simeq 10^6 \text{ g cm}^{-3}$  (right-hand figure) and maximum value of  $\sim 0.87$  and  $0.88$  for baryon density  $10^6 \text{ g cm}^{-3}$  (left-hand figure) and  $10^{4.5} \text{ g cm}^{-3}$ , therefore resonant oscillations are only allowed when the number density of protons is at least slightly larger than that of neutrons  $n_p \geq n_n$ . This result could make evident the de-neutralization process in the fireball, where  $Y_e$  is less than 0.5 at early times (there are not resonant oscillations) and after it becomes larger than 0.5 ( $Y_e > 0.5$ ) at latter times (resonant oscillations are allowed). Other important characteristic from the figures below is that the number density of neutrons to protons ratio  $n_n/n_p = (1-Y_e)/Y_e$  is lower for that fireball with more magnetization. For instance, taking the value of density  $\rho=10^6 \text{ g cm}^{-3}$

in both plots, we can see that  $Y_e(n_n/n_p) = 0.865(0.156)$  for  $B=50 B_c$  and  $Y_e(n_n/n_p) = 0.751(0.332)$  for  $B=0.1 B_c$ .

## 4.2. Phase of acceleration

When fireball starts expanding, the optically thick hot plasma expands with an increasing bulk Lorentz factor  $\Gamma \propto r/r_0$  with radius  $r$  following the adiabatic law and the comoving temperature drops as  $T'(r) \propto r_0/r$ . Protons and neutrons keep coupled with each other until the value of Lorentz factor  $\Gamma \simeq \eta$  is larger or less than a critical value or the dynamical time  $t'_{np} \simeq (r/\Gamma)$  is shorter than the elastic scattering time  $t'_{np} \simeq (n'_p \sigma_{np})^{-1}$  where  $\sigma_{np} \approx 3 \times 10^{-26} \text{ cm}^2$  and  $n'_p$  is the number density of protons. The critical value is defined by

$$\eta_\nu = \left( \frac{L \sigma_{np} Y_e}{4\pi m_p r_0} \right)^{1/4} \simeq 4.6 \times 10^2 L_{52}^{1/4} r_{07}^{-1/4} Y_e^{1/4}, \quad (66)$$

and depending on both  $\eta$  and  $\eta_\nu$  different processes of decoupling in a p-n outflow take place leading to varied energy ranges of neutrinos. For  $\eta \geq \eta_\nu$ , neutrinos in the energy range 5-10 GeV are expected (Bahcall & meszaros 2000) whereas  $\eta \leq \eta_\nu$  neutrino energy lies in the range 2-25 GeV or higher depending on the value of  $\eta$  (Meszaros & Rees 2000). Supposing that the decoupling takes place before coasting, then a slowly hadron shell ( $\Gamma_s$ ) is overtaken by another shell that in principle moves faster ( $\Gamma_f = \eta$ ) in the flow, producing inelastic collisions at  $r \sim 2\Gamma_s^2 r_0$ . Following Meszaros & Rees (2000), the typical collision Lorentz factor can be written as  $\Gamma_{rel} \simeq 1/2(\Gamma/\Gamma_s + \Gamma_s/\Gamma)$  with  $\Gamma = \sqrt{\Gamma_s \Gamma_f} \simeq \Gamma_f$ , and the total energy is  $E_{Tot} = (2m_p^2 + 2m_p E_{rel})^{1/2}$ , whereas the CM threshold energy is  $2m_p + m_\pi \sim 2 \text{ GeV}$ . Hence, at depths  $\tau_{pn} \sim n'_p \sigma_{pn} (r/\Gamma) > 1$  each neutron heated by an individual shock could collide  $k_\pi \sim 2$  times before its CM energy becomes less than threshold. From the total number of neutrons involved in the shock  $N_n = (1 - Y_e)\epsilon E/(\eta m_p)$  and assuming a shock dissipation efficiency

$\epsilon = 0.2$ , the number of expected events per year in DeepCore detector is

$$R_{\nu\bar{\nu}} \sim 0.33 E_{53} (1 - Y_e) N_{t37.7} R_{b3} h_{65}^2 \left( \frac{2 - \sqrt{2}}{1 + z - \sqrt{1 + z}} \right)^2 \text{ year}^{-1}, \quad (67)$$

where we have taken into account the product of density of ice and the effective volume at 100 GeV as  $\rho_{ice} V_{eff} \simeq 50$  megaton ( $V_{eff} \simeq 5.56 \times 10^{-2} \text{ km}^3$  and target protons of  $N_t \sim 10^{37.7} N_{t37.7}$ ) (Abbasi & et al. 2012a), a burst rate out to a Hubble radius of  $10^3 R_b$  and an Einstein-de-Sitter universe with Hubble constant  $H=65 h_{65} \text{ km/s/Mpc}$ . Recently, Murase et al. (2013) showed that for a low-luminosity GRBs at  $D=10 \text{ Mpc}$  with  $\Gamma=10$  and neutron luminosity  $L_n = 2 \times 10^{46} \text{ erg/sec}$ , quasi-thermal neutrinos around 10 GeV are expected in DeepCore.

We plot the critical Lorentz factors ( $\eta_\nu$ ) (figure above) and number of neutrinos ( $R_{\nu\nu}$ ) (figure below) as a function of temperature when the resonance and charged-neutrality conditions are satisfied as shown in fig. 12. From figures above, one can see that the critical Lorentz factor is a decreasing function of temperature and decreasing function of baryon density. Besides,  $\eta_\nu$  lies in the range  $\simeq 436$  to  $662$  for temperatures and baryon densities in the range  $50$  to  $10^3 \text{ keV}$  and  $10$  to  $10^3 (1 \text{ to } 10^2) \text{ g/cm}^3$  for  $B=10^{-4.5} B_c$  ( $B=10^{-5.5} B_c$ ). From these plots can be seen that the effect of magnetic field is to increase  $\eta_\nu$ . In figures below, we can observe that the number of expected neutrinos is an increasing function of temperature and baryon density. Comparing both plots we see that as  $\rho = 10^2 \text{ g/cm}^3$  and  $T=0.7 \text{ MeV}$  the expected neutrinos are  $\simeq 2.23$  for  $B=10^{-4.3} B_c$  and  $\simeq 4$  for  $B=10^{-5.3} B_c$ , which tells us that the effect of magnetic field is to decrease the number of expected neutrinos. From that we can see the importance of knowing the strength of magnetic field and as it could alter the dynamics.

It is important to notice that independently of the model and considerations assumed (Bahcall & Meszaros 2000; Meszaros & Rees 2000; Murase et al. 2013) multi-GeV neutrinos are expected on Earth, hence we will estimate their flavor ratio. Considering the flux ratio for  $\pi^\pm$  and  $\mu^\pm$  decay as  $\dot{N}_{\nu_\mu} \simeq \dot{N}_{\bar{\nu}_\mu} \simeq 2\dot{N}_{\nu_e} \simeq 2\dot{N}_{\bar{\nu}_e}$  (Razzaque & Meszaros 2006a) and from oscillation probabilities at  $10^{11} \text{ cm}$ ,  $10^{12} \text{ cm}$  and  $10^{13} \text{ cm}$  given in the section III, we compute the flavor ratio for neutrino energies of 5, 10, 20 and 50 GeV as shown in table 3. From this table one can see an interesting

result, that although tau neutrino at GeV energies is not created by p-n decoupling, it appears due to the resonant oscillations of muon neutrinos in the fireball. In addition, from table 3 and eq. 58 we estimate the flavor ratio expected on Earth for the same range of neutrino energy as shown in table 4. As shown in this table, GeV sub-photospheric neutrinos with deviations of this standard flavor ratios are expected.

## 5. Results and Conclusions

We have explicitly calculated the neutrino self-energy and neutrino effective potential up to an order  $m_W^{-4}$  as a function of temperature, chemical potential, magnetic field and neutrino energy. We have shown that for neutrinos in the GeV energy range as well as small chemical potentials which is the case of solar and atmospheric neutrino parameter (small particle-antiparticle asymmetry) the contribution of  $m_W^{-4}$  terms to the neutrino effective potential is relevant. In the magnetic field framework, we have derived it at the strong, moderate (above and below  $B_c$ ) and weak-field limit, taking into account a background composed of pairs  $e^\pm$ , photons, protons, neutrons and neutrinos. Also we have derived the resonance and charged-neutrality conditions. By considering the neutrino effective potential at the moderate field limit, using the typical values of a magnetized GRB fireball and requiring the resonance and charged-neutrality conditions, we have studied the thermal and quasi-thermal neutrino oscillations assuming a neutron abundance which is comparable to that of protons. In the fireball scenario, thermal neutrino oscillations have been studied at the initial stage ( $r_0 \simeq 10^{6.5} - 10^{7.5}$  cm,  $B \simeq 0.1 - 50 B_c$  and  $T \simeq 1 - 10$ ) MeV whereas quasi-thermal GeV neutrinos in the phase of acceleration ( $r \simeq 10^{11} - 10^{13}$  cm,  $B \simeq 10^{-4.3} - 10^{-5.3} B_c$  and  $T \simeq 50 - 700$  keV). This complete analysis has been carried out using the two- (solar, atmospheric and accelerator neutrino parameters) and three-neutrino mixing.

The results for the initial stage are:

$E_\nu$ (GeV)	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ ( $r=10^{11}$ cm)	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ ( $r=10^{12}$ cm)	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ ( $r=10^{13}$ cm)
5	1.000:1.959:0.041	1.000:0.021:1.979	1.000:0.848:1.152
10	1.000:0.164:1.837	1.000:0.951:1.049	1.000:0.000:2.000
20	1.000:1.101:0.899	1.000:0.282:1.718	1.000:1.917:0.083
50	1.000:0.675:1.325	1.000:0.002:1.998	1.000:1.839:0.162

Table 3: The flavor ratio on the surface of the fireball for four neutrino energies ( $E_\nu=5$  GeV, 10 GeV, 20 GeV and 50 GeV), leaving the star to three distances  $r=10^{11}$  cm,  $10^{12}$  cm and  $10^{13}$  cm.

$E_\nu$ (TeV)	$\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$ ( $r=10^{11}$ cm)	$\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$ ( $r=10^{12}$ cm)	$\phi_{\nu_e}^0 : \phi_{\nu_\mu}^0 : \phi_{\nu_\tau}^0$ ( $r=10^{13}$ cm)
5	1.063:0.998:0.939	0.936:1.002:1.062	0.990:1.000:1.010
10	0.945:1.001:1.053	0.997:1.000:1.003	0.935:1.002:1.064
20	1.007:0.999:0.994	0.953:1.001:1.046	1.060:0.999:0.941
50	0.979:1.001:1.021	0.935:1.002:1.064	1.055:0.999:0.947

Table 4: The flavor ratio expected on Earth for four neutrino energies ( $E_\nu=5$  GeV, 10 GeV, 20 GeV and 50 GeV), leaving the star to three distances  $r=10^{11}$  cm,  $10^{12}$  cm and  $10^{13}$  cm.

1. Resonant oscillations are suppressed for baryon densities greater than  $\sim 10^8 \text{ g cm}^{-3}$  for  $50 B_c$  and  $\sim 10^6 \text{ g cm}^{-3}$  for  $0.1 B_c$ .
2. Neutrinos can hardly oscillate resonantly for a fireball with number density of neutrons greater than protons  $n_n \geq n_p$  or for proton-to-baryon ratio larger than 0.52 ( $Y_e \leq 0.52$ ).
3. The number density of neutrons to protons ratio ( $n_n/n_p$ ) is lower for that fireball with more magnetization. It is due to positron capture on neutrons is greatly accelerated by the large magnetic phase space factor (Thompson & Gill 2013). Also baryon densities are larger for a fireball with more magnetization.

From the phase of acceleration, we showed that:

1. The critical Lorentz factor for neutrino production is limited by the load density, temperature and the magnetic field. The effect of magnetic field in the emission region is to decrease the expected number of neutrinos.
2. GeV neutrinos created in the sub-photospheric region can oscillate resonantly. Due to this, we estimate the neutrino flavor ratio and deviations of standard flavor ratios are expected.

Neutrinos of energies of 1 to 30 MeV are very similar to those produced by type II supernova i.e, SN1987A, however are of cosmological distance. These cosmological events make the thermal neutrino flux very low on Earth compared to the ones we had seen from the supernova SN1987A.



Although with the current neutrino telescopes, low energy neutrinos would be pretty difficult to detect, such survey would help us to understand the dynamics of the jet as it changes with the content of baryons.

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### A. Strong Magnetic field: $\Omega_B \gg 1$

In the strong magnetic field approximation, the energy of charged particles is modified confining the particles to the lowest Landau level ( $n = 0$ ). Thus, the number density of electrons given by Eq. (23) will become

$$n_e^0 = \frac{eB}{2\pi^2} \int_0^\infty dp_3 f_{e,0}, \quad (\text{A1})$$

where

$$f(E_{e,0}) = \frac{1}{e^{\beta(E_{e,0}-\mu)} + 1}, \quad (\text{A2})$$

and the electron energy in the lowest Landau level is,

$$E_{e,0}^2 = (p_3^2 + m_e^2). \quad (\text{A3})$$

Assuming that the chemical potentials ( $\mu$ ) of the electrons and positrons are smaller than their energies ( $\mu \leq E_e$ ), the fermion distribution function can be written as a sum given by

$$f(E_{e,0}) = \frac{1}{e^{\beta(E_{e,0}-\mu)} + 1} \approx \sum_{l=0}^{\infty} (-1)^l e^{-\beta(E_{e,0}-\mu)(l+1)}. \quad (\text{A4})$$

Therefore, the Lorentz scalars in this approximation are reduced to (Elizalde et al. 2004; Sahu et al. 2009a)

$$b_W = \sqrt{2}G_F \left[ \left( 1 + \frac{3}{2} \frac{m_e^2}{m_W^2} + \frac{eB}{m_W^2} + \frac{E_{\nu_e} k_3}{m_W^2} + \frac{E_{\nu_e}^2}{m_W^2} \right) (N_e^0 - \bar{N}_e^0) - \frac{eB}{2\pi^2 m_W^2} \int_0^\infty dp_3 \left\{ 2k_3 E_{e,0} + 2E_{\nu_e} \left( E_{e,0} - \frac{m_e^2}{2E_{e,0}} \right) \right\} (f_{e,0} + \bar{f}_{e,0}) \right] \quad (\text{A5})$$

and

$$c_W = \sqrt{2}G_F \left[ \left( 1 + \frac{1}{2} \frac{m_e^2}{m_W^2} + \frac{eB}{m_W^2} - \frac{E_{\nu_e} k_3}{m_W^2} - \frac{k_3^2}{m_W^2} \right) (N_e^0 - \bar{N}_e^0) - \frac{eB}{2\pi^2 m_W^2} \int_0^\infty dp_3 \left\{ 2E_{\nu_e} \left( E_{e,0} - \frac{m_e^2}{2E_{e,0}} \right) \right\} + 2k_3 \left( E_{e,0} - \frac{3m_e^2}{2E_{e,0}} \right) \right\} (f_{e,0} + \bar{f}_{e,0}) \right] \quad (\text{A6})$$

For Z-exchange diagram we do not have magnetic contribution and for the tadpole diagram only electron loop will be affected by the magnetic field (Sahu et al. 2009b). The electron number



density and other useful quantities at the strong field limit are

$$N_e^0 = \frac{m_e^3}{2\pi^2} \frac{B}{B_c} \sum_{l=0}^{\infty} (-1)^l e_l^\alpha K_1(\sigma_l), \quad (\text{A7})$$

$$N_e^0 - \bar{N}_e^0 = \frac{B}{B_c} \frac{m_e^3}{\pi^2} \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l K_1(\sigma_l), \quad (\text{A8})$$

$$\begin{aligned} \frac{eB}{2\pi^2} \int_0^\infty dp_3 E_0(f_{e,0} + \bar{f}_{e,0}) &= \frac{m_e^4}{\pi^2} \frac{B}{B_c} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left( K_0(\sigma_l) + \frac{K_1(\sigma_l)}{\sigma_l} \right), \\ \frac{eB}{2\pi^2} \int_0^\infty dp_3 \frac{1}{E_0}(f_{e,0} + \bar{f}_{e,0}) &= \frac{m_e^4}{\pi^2} \frac{B}{B_c} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l K_0(\sigma_l), \end{aligned} \quad (\text{A9})$$

where we have defined

$$\alpha_l = \beta\mu(l+1) \quad \text{and} \quad \sigma_l = \beta m_e(l+1) \quad (\text{A10})$$

with  $K_i$  is the modified Bessel function of integral order  $i$ . Replacing Eqs. (A8) and (A9) in the Lorentz scalars (Eqs. A5 and A6), we obtain

$$\begin{aligned} b_W &= \frac{\sqrt{2}G_F m_e^3}{\pi^2} \frac{B}{B_c} \left[ \left\{ 1 + \frac{m_e^2}{m_W^2} \left( \frac{3}{2} + 2 \frac{E_{\nu,e}^2}{m_e^2} + \frac{B}{B_c} \right) \right\} \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l K_1(\sigma_l) \right. \\ &\quad \left. - \frac{m_e^2}{m_W^2} \frac{E_{\nu,e}}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ 3K_0(\sigma_l) + 4 \frac{K_1(\sigma_l)}{\sigma_l} \right\} \right], \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} c_W &= \frac{\sqrt{2}G_F m_e^3}{\pi^2} \frac{B}{B_c} \left[ \left\{ 1 + \frac{m_e^2}{m_W^2} \left( \frac{1}{2} - 2 \frac{E_{\nu,e}^2}{m_e^2} + \frac{B}{B_c} \right) \right\} \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l K_1(\sigma_l) \right. \\ &\quad \left. - 4 \frac{m_e^2}{m_W^2} \frac{E_{\nu,e}}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \frac{K_1(\sigma_l)}{\sigma_l} \right]. \end{aligned} \quad (\text{A12})$$

Finally, from Eqs. (6), (A11) and (A12) and for neutrinos moving along the direction of magnetic field, the neutrino effective potential in the strong magnetic field regime is

$$\begin{aligned} V_{eff} &= \frac{\sqrt{2}G_F m_e^3}{\pi^2} \frac{B}{B_c} \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l K_1(\sigma_l) \left\{ 1 + \frac{m_e^2}{m_W^2} \left( \frac{3}{2} + 2 \frac{E_\nu^2}{m_e^2} + \frac{B}{B_c} \right) \right. \right. \\ &\quad \left. \left. - \left( 1 + \frac{m_e^2}{m_W^2} \left( \frac{1}{2} - 2 \frac{E_\nu^2}{m_e^2} + \frac{B}{B_c} \right) \right) \cos \phi \right\} \right. \\ &\quad \left. - 4 \frac{m_e^2}{m_W^2} \frac{E_\nu}{m_e} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ \frac{3}{4} K_0(\sigma_l) + \frac{K_1(\sigma_l)}{\sigma_l} - \frac{K_1(\sigma_l)}{\sigma_l} \cos \phi \right\} \right]. \end{aligned} \quad (\text{A13})$$

Doing  $\Omega_B = B/B_c$  and the previous effective potential is written as shown in eq. (29).

**B. Moderate Magnetic field:  $\Omega_B > 1$  and  $\Omega_B \leq 1$**

The electron energy in the magnetic field is given by

$$E_{e,n}^2 = (p_3^2 + m_e^2 + 2neB) = p_3^2 + m_e^2(1 + 2n\Omega_B), \quad (\text{B1})$$

In this case, the number density of electrons (eq. 23) is written as

$$n_e = \frac{\Omega_B m_e^2}{2\pi^2} \left[ \int_0^\infty dp_3 f_{e,0} + 2 \sum_{n=1}^\infty \int_0^\infty dp_3 f_{e,n} \right], \quad (\text{B2})$$

and the electron distribution function by

$$f(E_{e,n}) = \frac{1}{e^{\beta(E_{e,n}-\mu)} + 1} \approx \sum_{l=0}^\infty (-1)^l e^{-\beta(E_{e,n}-\mu)(l+1)}. \quad (\text{B3})$$

Calculating the below useful quantities in this regime,

$$n_e - \bar{n}_e = \frac{m^3}{\pi^2} \Omega_B \left[ \sum_{l=0}^\infty (-1)^l \sinh \alpha_l \left\{ K_1(\sigma_l) + 2 \sum_{n=1}^\infty \lambda_n K_1(\sigma_l \lambda_n) \right\} \right] \quad (\text{B4})$$

$$n_e + \bar{n}_e = \frac{m^3}{\pi^2} \Omega_B \left[ \sum_{l=0}^\infty (-1)^l \cosh \alpha_l \left\{ K_1(\sigma_l) + 2 \sum_{n=1}^\infty \lambda_n K_1(\sigma_l \lambda_n) \right\} \right] \quad (\text{B5})$$

$$\int_n^\infty dp_3 E_n(f_{e,n} + \bar{f}_{e,0}) = 2m_e^2 \sum_{l=0}^\infty (-1)^l \cosh \alpha_l \left[ \left( K_0(\sigma_l) + \frac{K_1(\sigma_l)}{\sigma_l} \right) + 2 \sum_{n=1}^\infty \lambda_n^2 \left( K_0(\sigma_l \lambda_n) + \frac{K_1(\sigma_l \lambda_n)}{\sigma_l} \right) \right], \quad (\text{B6})$$

$$\int_0^\infty dp_3 \frac{1}{E_n} (f_{e,n} + \bar{f}_{e,n}) = 2 \sum_{l=0}^\infty (-1)^l \cosh \alpha_l \left[ K_0(\sigma_l) + \frac{1}{2} \sum_{n=1}^\infty K_0(\sigma_l \lambda_n) \right], \quad (\text{B7})$$

where  $\lambda$  is defined by

$$\lambda^2 = \begin{cases} 2n\Omega_B & \text{for } \Omega_B > 1 & \text{moderately above} \\ 1 + 2n\Omega_B & \text{for } \Omega_B \leq 1 & \text{moderately below,} \end{cases} \quad (\text{B8})$$

with the Lorentz scalars and again with  $A_e = \sqrt{2}G_F \frac{m_e^3 \Omega_B}{\pi^2}$ , the neutrino effective potential in the moderate regimen can be written as shown in eq. (30).

### C. Weak Magnetic field: $\Omega_B \ll 1$

In this regimen, sums over the parameter  $\lambda = \sqrt{1 + 2n\Omega_B}$  can be approximated by an integral  $\sum_n \rightarrow \int dn$ . Therefore, it is useful to write the following relations

$$\begin{aligned} \sum_{n=0}^{\infty} \lambda_n^2 K_0(\sigma_l \lambda_n) &= \Omega_B^{-1} \left[ 2 \frac{K_0(\sigma_l)}{\sigma_l} + \left( 1 + \frac{4}{\sigma_l^2} \right) \frac{K_1(\sigma_l)}{\sigma_l} \right] \\ \sum_{n=0}^{\infty} \lambda_n K_1(\sigma_l \lambda_n) &= \Omega_B^{-1} \left[ \frac{K_0(\sigma_l)}{\sigma_l} + \frac{2}{\sigma_l^2} K_1(\sigma_l) \right], \end{aligned} \quad (\text{C1})$$

where we have applied the integrals

$$\begin{aligned} \int_1^{\infty} \frac{t^2 dt}{\sqrt{t^2 - 1}} e^{-\sigma_l t} &= K_0(\sigma_l) + \frac{K_1(\sigma_l)}{\sigma_l} \\ \int_1^{\infty} \frac{t dt}{\sqrt{t^2 - 1}} e^{-\sigma_l t} &= K_1(\sigma_l) \\ \int_1^{\infty} \frac{dt}{\sqrt{t^2 - 1}} e^{-\sigma_l t} &= K_0(\sigma_l) \end{aligned} \quad (\text{C2})$$

for  $\text{Re } \sigma_l > 0$  and we have used the recurrence relation  $K_3(\sigma_l) = \frac{4}{\sigma_l} K_0(\sigma_l) + \left( 1 + \frac{8}{\sigma_l^2} \right) K_1(\sigma_l)$  and  $K_2(\sigma_l) = K_0(\sigma_l) + \frac{2}{\sigma_l} K_1(\sigma_l)$ . Therefore, from eqs. (B4), (B5) and (30), the neutrino effective potential is given by eq. (31) and useful functions of electron number density can be written as

$$n_e - \bar{n}_e = \frac{m^3}{\pi^2} \Omega_B \left[ 2 \sum_{l=0}^{\infty} (-1)^l \sinh \alpha_l \left\{ \left( \frac{K_0(\sigma_l)}{\sigma_l} + \frac{K_1(\sigma_l)}{\sigma_l^2} \right) \Omega_B^{-1} - \frac{1}{2} K_1(\sigma_l) \right\} \right] \quad (\text{C3})$$

$$n_e + \bar{n}_e = \frac{m^3}{\pi^2} \Omega_B \left[ 2 \sum_{l=0}^{\infty} (-1)^l \cosh \alpha_l \left\{ \left( \frac{K_0(\sigma_l)}{\sigma_l} + \frac{K_1(\sigma_l)}{\sigma_l^2} \right) \Omega_B^{-1} - \frac{1}{2} K_1(\sigma_l) \right\} \right]. \quad (\text{C4})$$

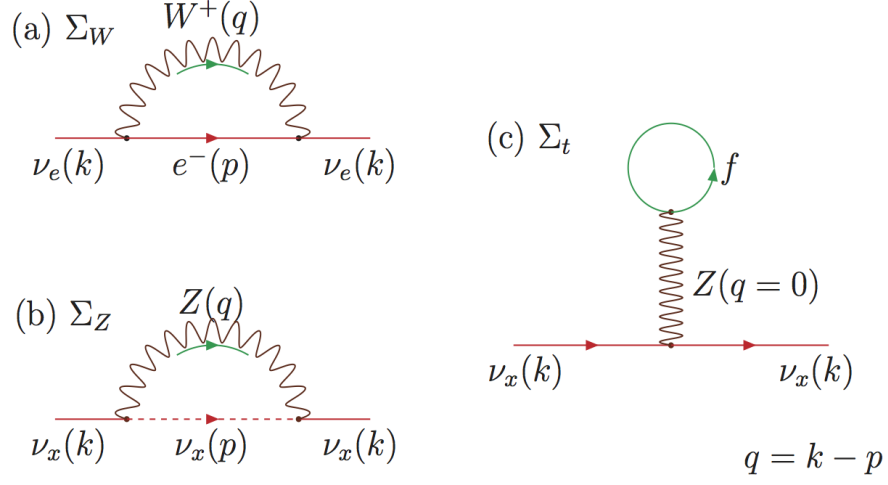


Fig. 1.— One-loop contribution to the neutrino self-energy in a magnetized medium. a) W-exchange diagram: The solid line represents the electron propagator and the wiggly line the W-boson in a magnetized medium. b) Z-exchange diagram: The dashed line corresponds to the neutrino propagator in a thermal medium and the wiggly line to the Z-boson c) Tadpole diagram: The solid line represents the fermion propagator and the wiggly line the Z-boson in a magnetized medium.

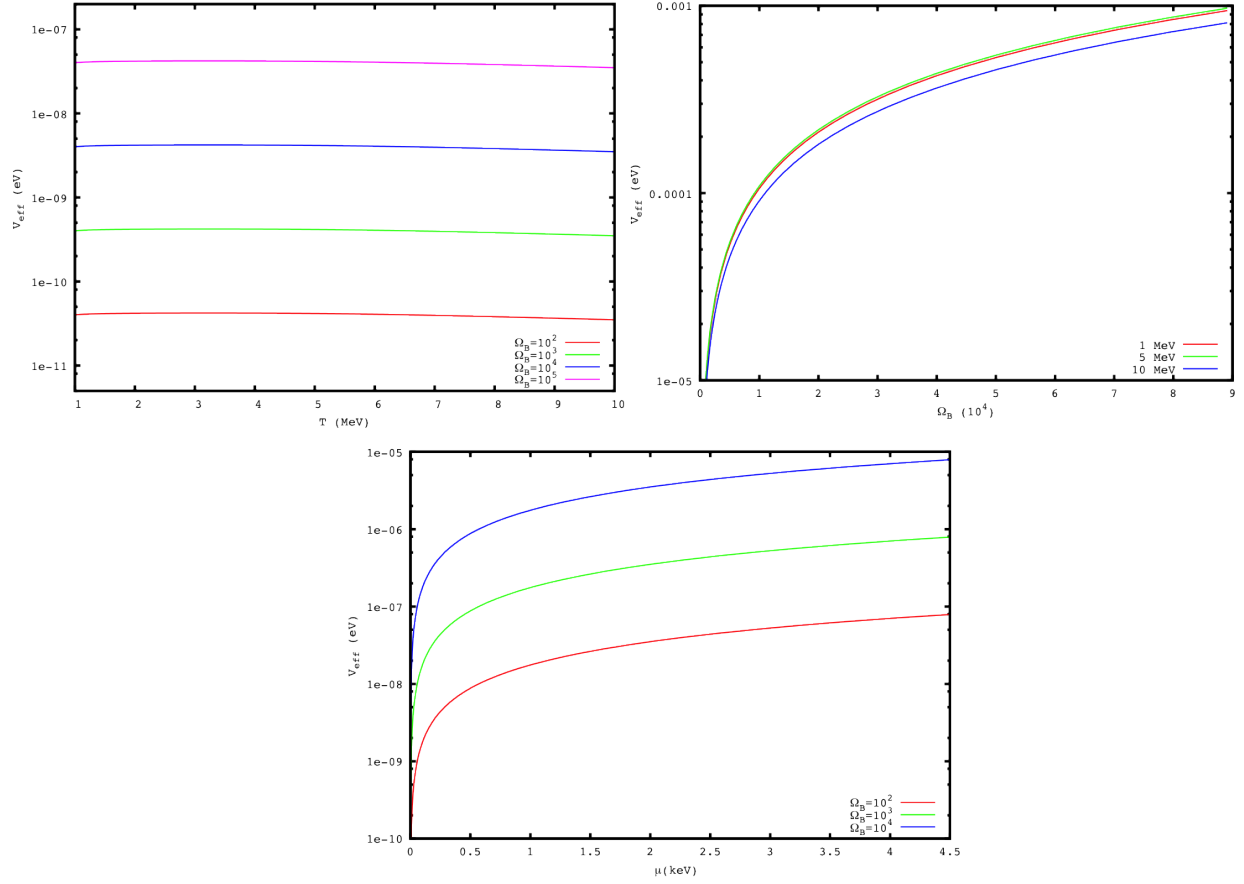


Fig. 2.— Neutrino effective potential in the strong magnetic field regime as a function of temperature ( $T$ ) (left-hand figure above), magnetic field ( $\Omega_B$ ) (right-hand figure above), chemical potential ( $\mu$ ) (figure below) and neutrino energy ( $E_\nu$ ). All plots are obtained for a neutrino energy of 10 MeV.

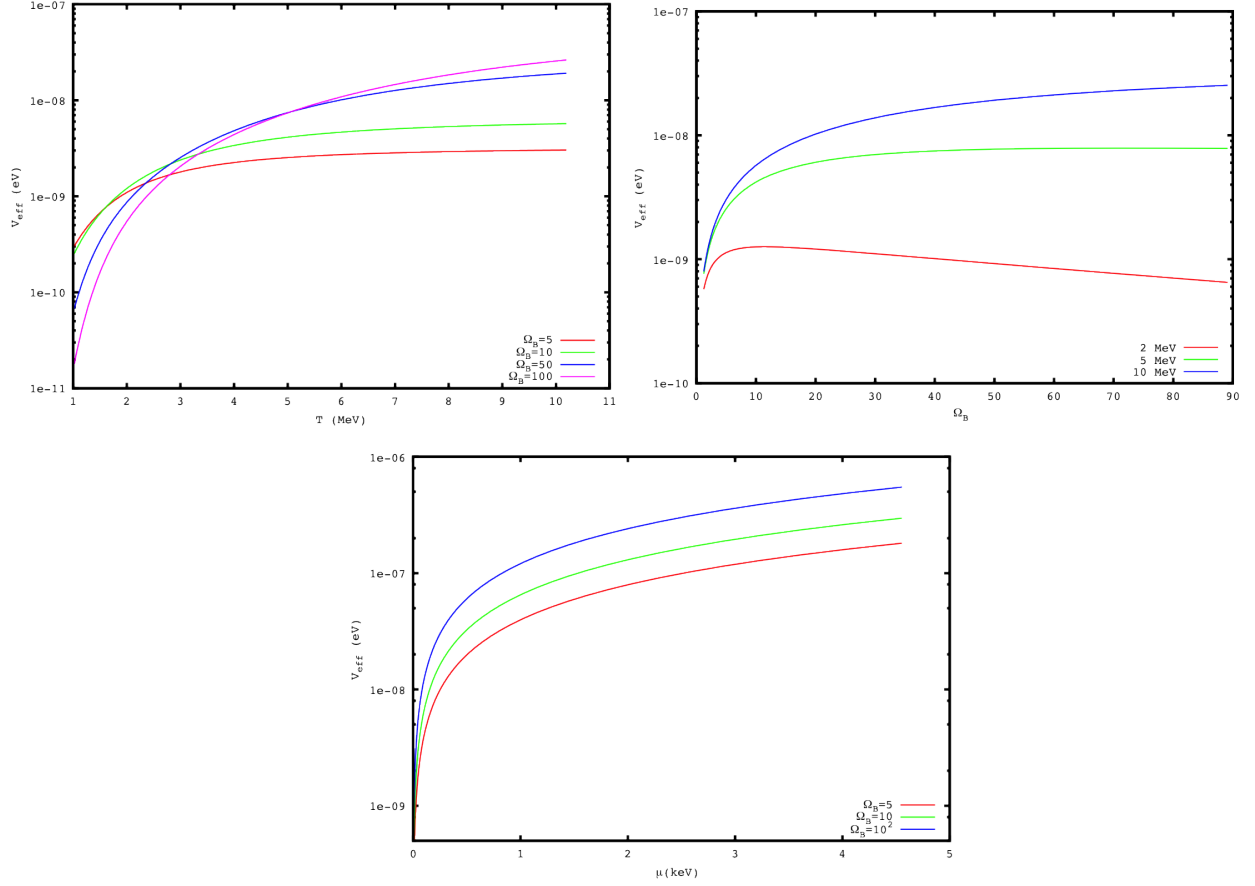


Fig. 3.— Neutrino effective potential in moderate above critical magnetic field regime as a function of temperature ( $T$ ) (left-hand figure above), magnetic field ( $\Omega_B$ ) (right-hand figure above), chemical potential ( $\mu$ ) (figure below) and neutrino energy ( $E_\nu$ ). All plots are obtained for a neutrino energy of 10 MeV.

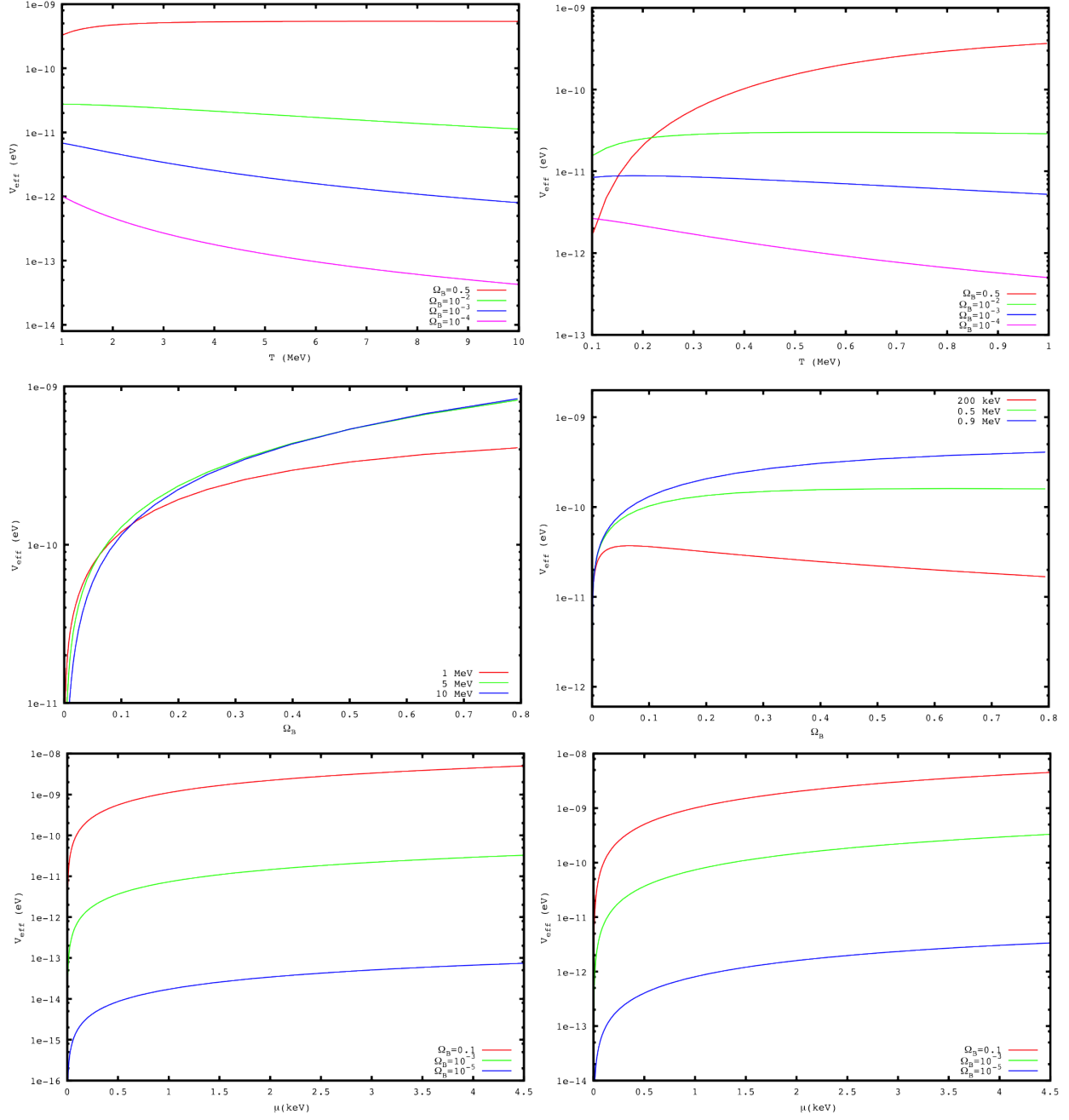


Fig. 4.— Neutrino effective potential in moderate below critical magnetic field regime as a function of temperature ( $T$ ) (top), magnetic field ( $\Omega_B$ ) (middle), chemical potential ( $\mu$ ) (bottom). In the left column a neutrino energy  $E_\nu = 10$  MeV was used whereas the right column was plotted for  $E_\nu = 10$  GeV.

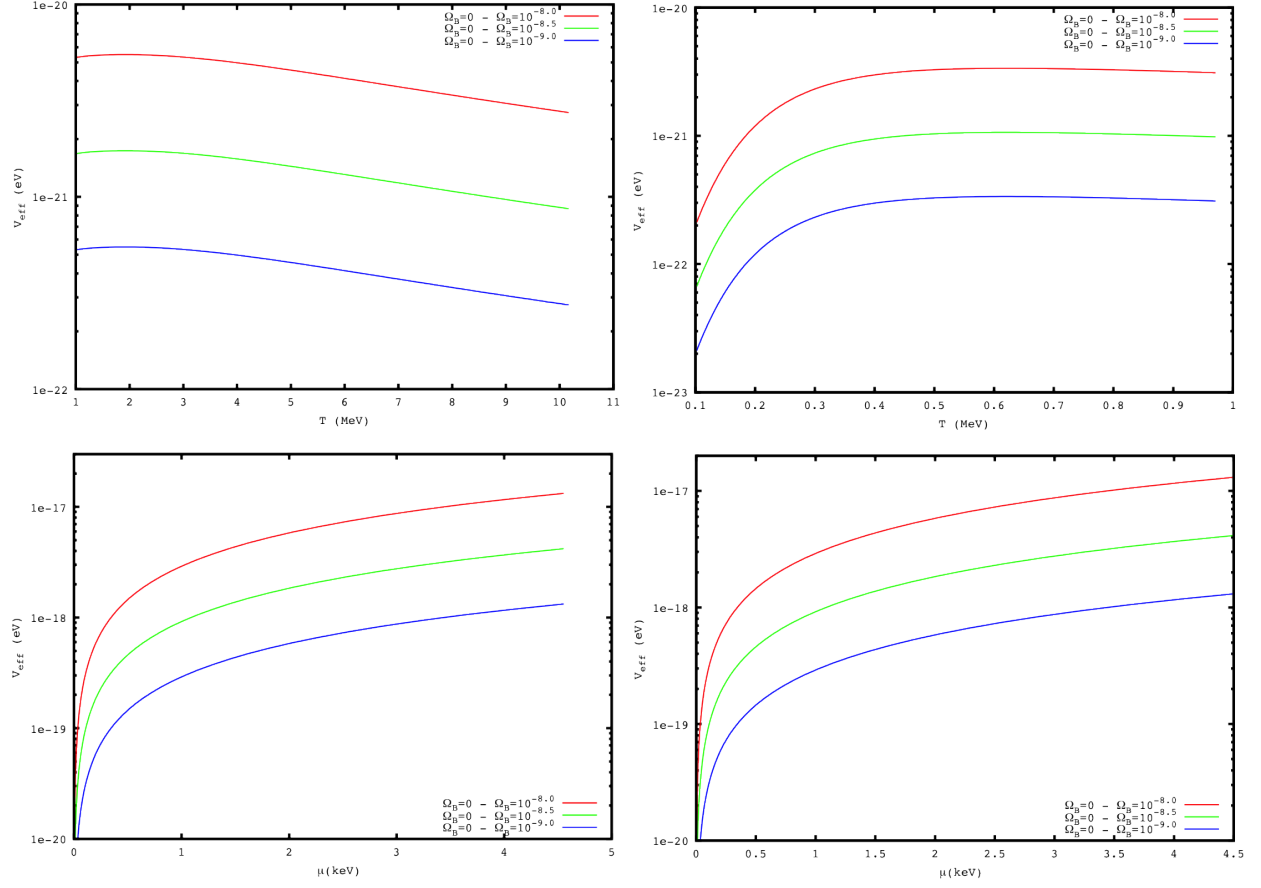


Fig. 5.— Neutrino effective potential in weak magnetic field regime as a function of temperature ( $T$ ) (above) and chemical potential ( $\mu$ ) (below). In the left column a neutrino energy  $E_\nu = 10$  MeV was used whereas the right column was plotted for  $E_\nu = 10$  GeV.



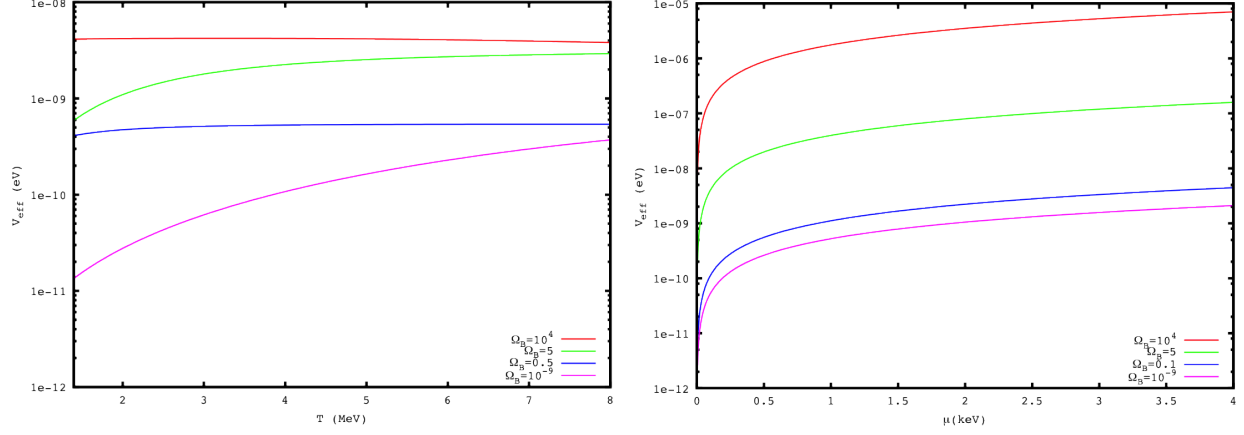


Fig. 6.— Neutrino effective potential as a function of temperature (left) and chemical potential (right) for fixed values of magnetic field in the strong ( $10^4 B_c$ ), moderate (5 and  $0.5 B_c$  for above and below, respectively) and weak ( $10^{-9} B_c$ ) regime.

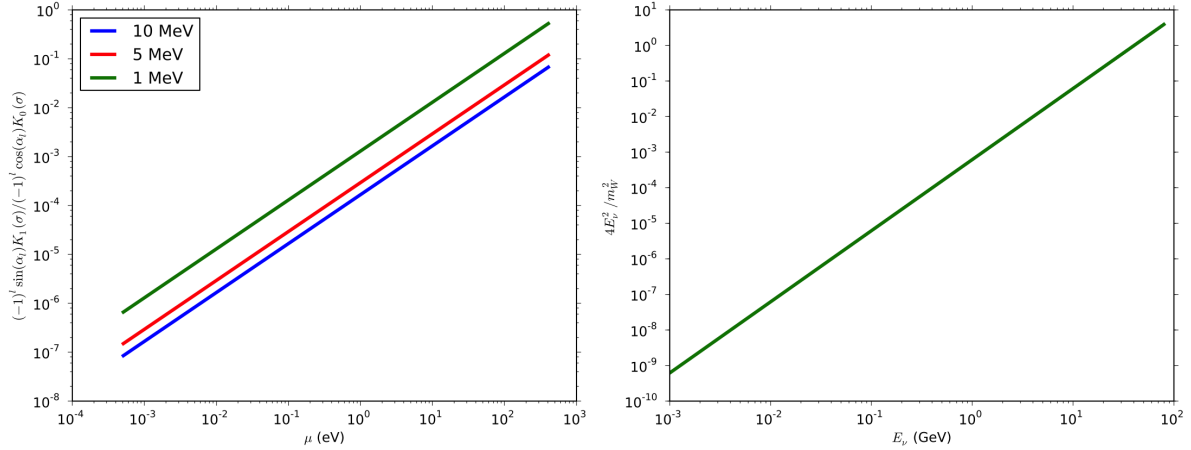


Fig. 7.— Contribution of the terms of order  $m_W^{-4}$  in the neutrino effective potential.

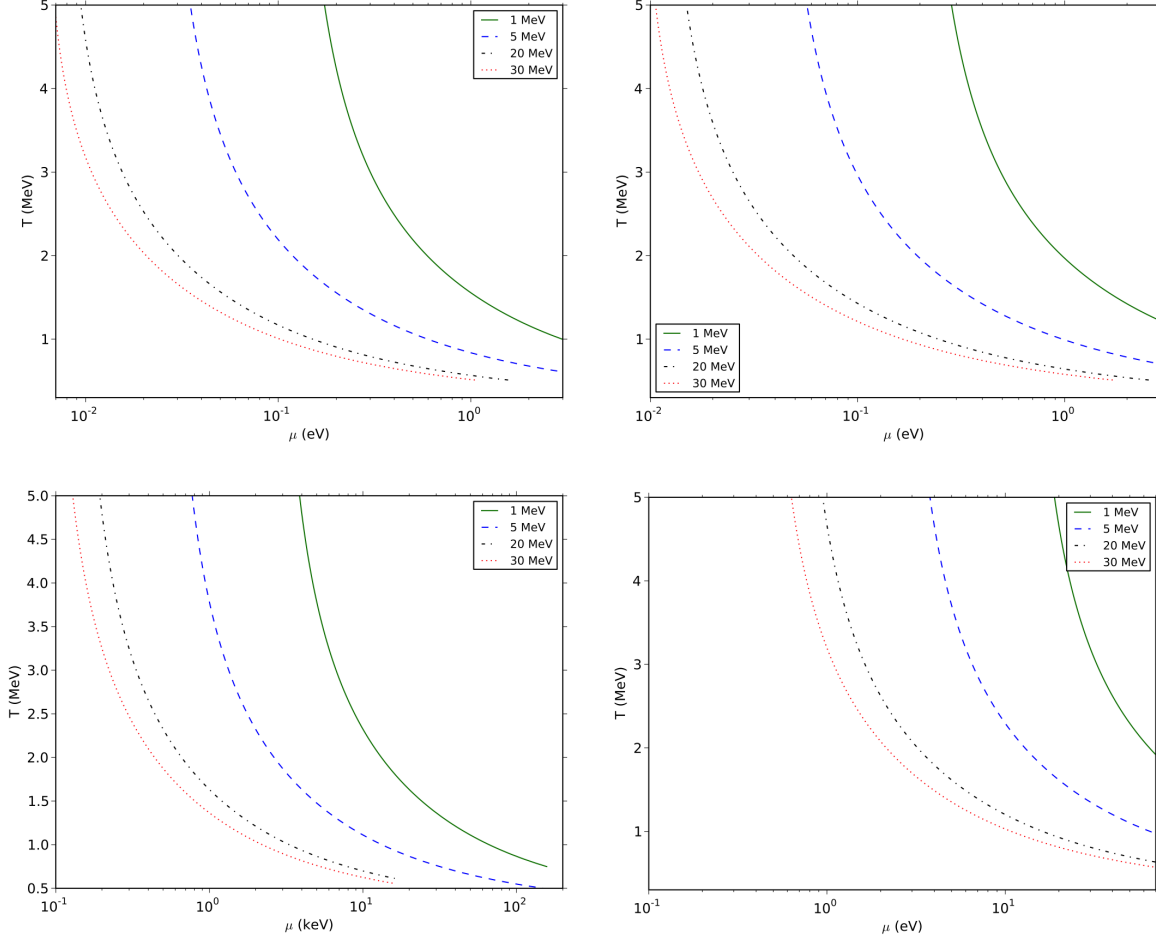


Fig. 8.— Contour plots of temperature ( $T$ ) and chemical potential ( $\mu$ ) as a function of thermal neutrino energy for which the resonance condition is satisfied. We have applied the neutrino effective potential with moderate magnetic field ( $\Omega_B = 10$ ) and used the best fit values of the two; solar (left-hand figure above), atmospheric (right-hand figure above), and accelerator (left-hand figure below), and three (left-hand figure below) neutrino mixing.

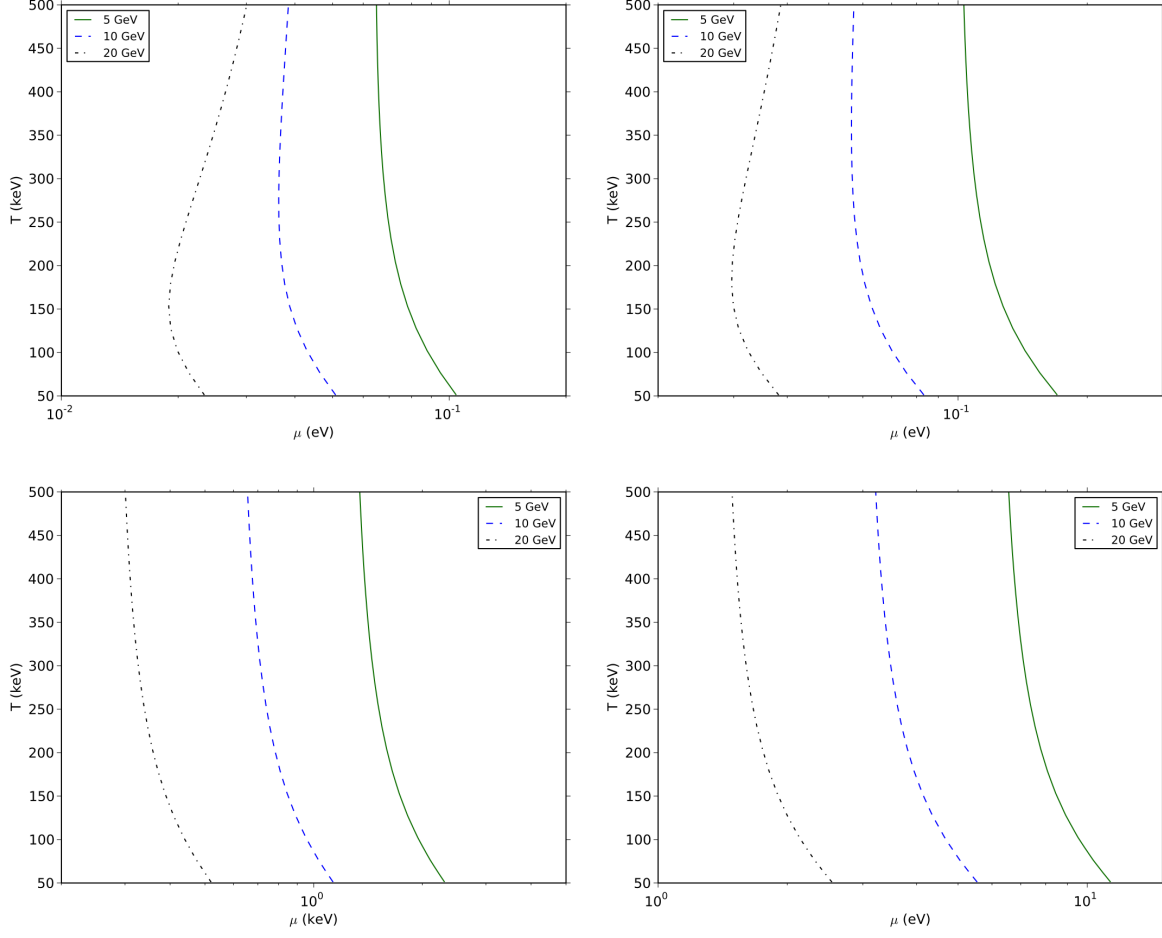


Fig. 9.— Contour plots of temperature ( $T$ ) and chemical potential ( $\mu$ ) as a function of quasi-thermal neutrino energy for which the resonance condition is satisfied. We have applied the neutrino effective potential with moderate magnetic field ( $\Omega_B = 10^{-4.3}$ ) and used the best fit values of the two; solar (left-hand figure above), atmospheric (right-hand figure above), and accelerator (left-hand figure below), and three (left-hand figure below) neutrino mixing.

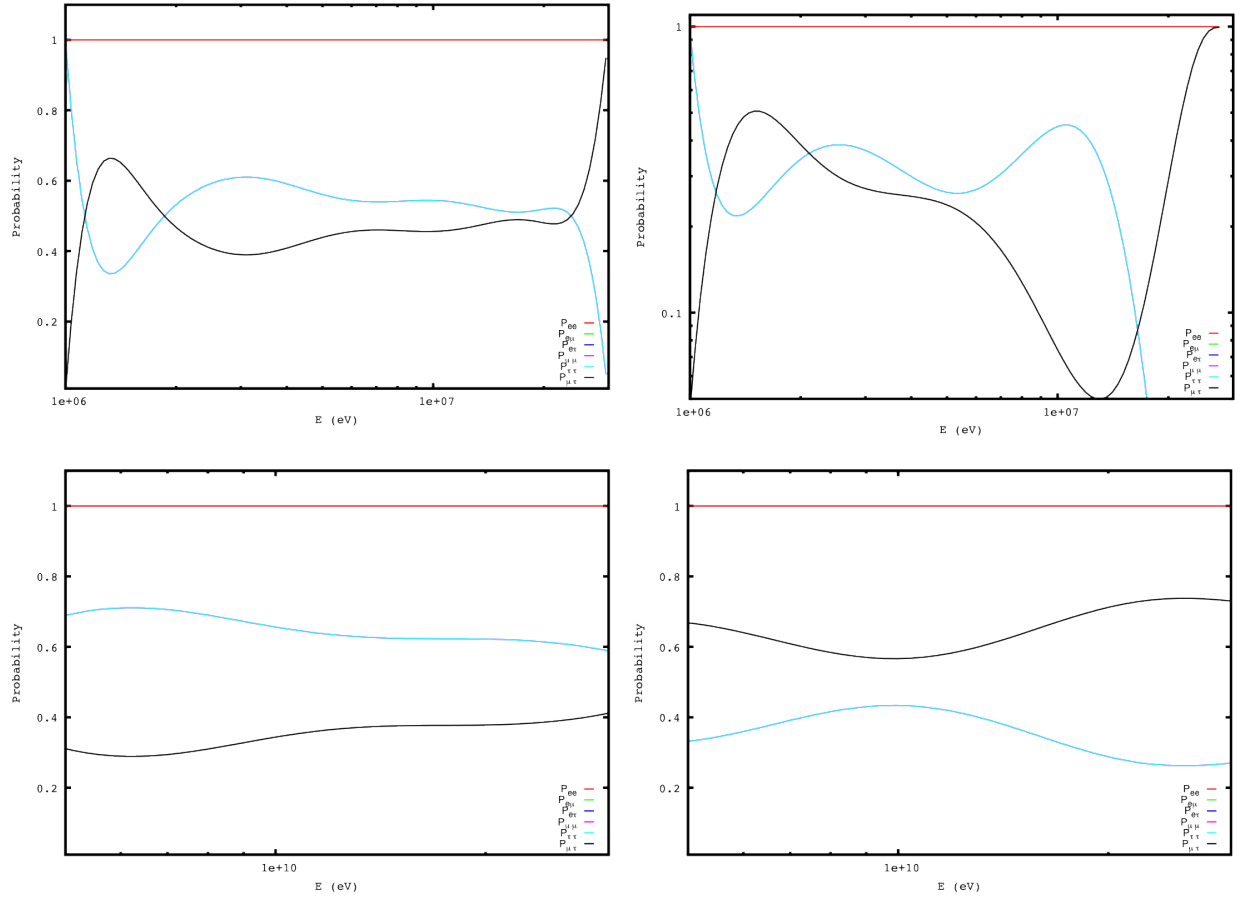


Fig. 10.— Oscillation probabilities of thermal MeV (figures above) and sub-photospheric (figures below) neutrinos as a function of neutrino energy are plotted. For thermal neutrinos we have used the values of magnetic field  $10 B_c$  and the fireball radii  $10^7$  cm (left figure) and  $10^8$  cm (right figure) whereas for sub-photospheric neutrinos we have used the values  $B=10^{-4.3} B_c$  and  $10^{11}$  cm (left figure) and  $10^{12}$  cm, respectively.

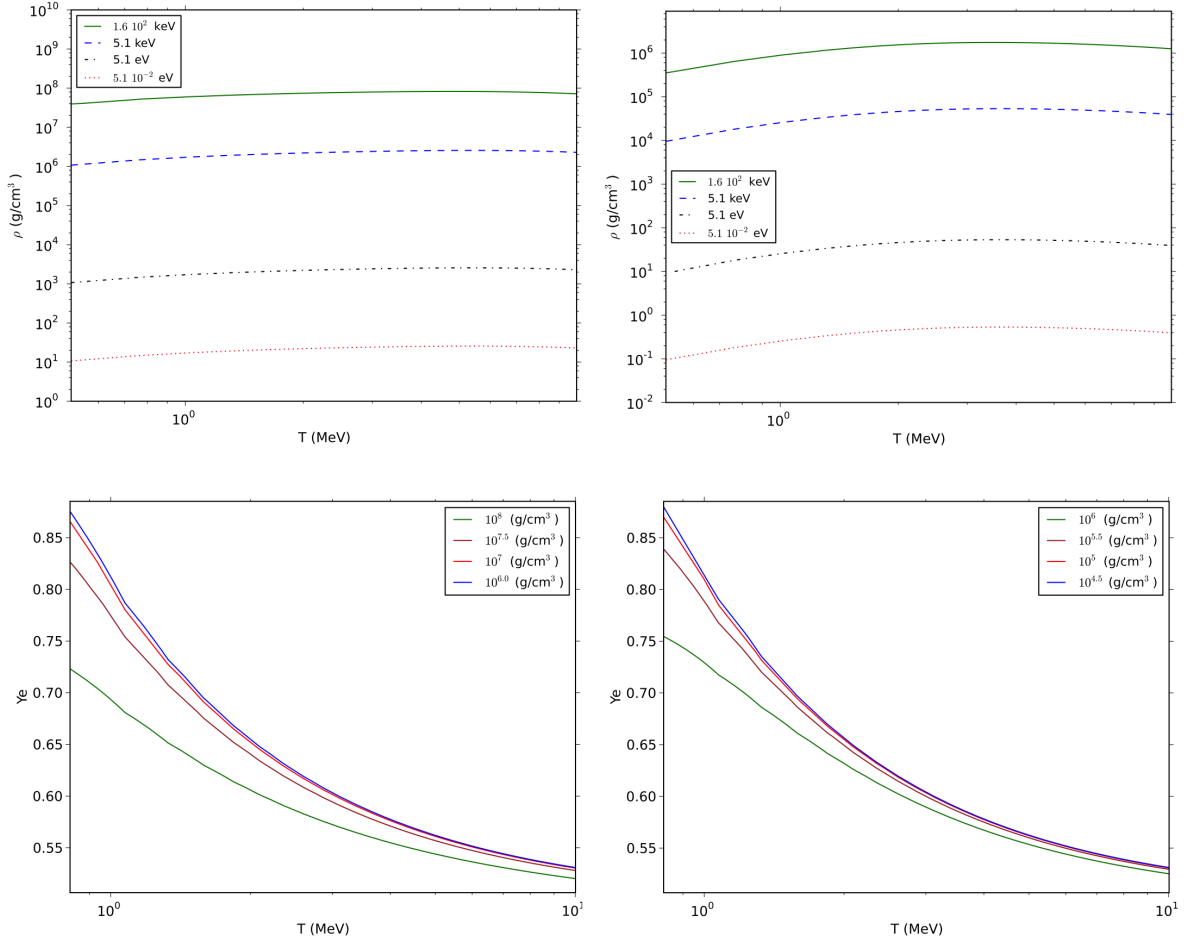


Fig. 11.— Contour plots of baryon density (above) and  $Y_e$  (below) as a function of temperature for which the equilibrium condition is established. We use the values of temperature and chemical potential in the resonance condition range and moderate field limit:  $50 B_c$  (left-hand figures) and  $0.1 B_c$  (right-hand figures).

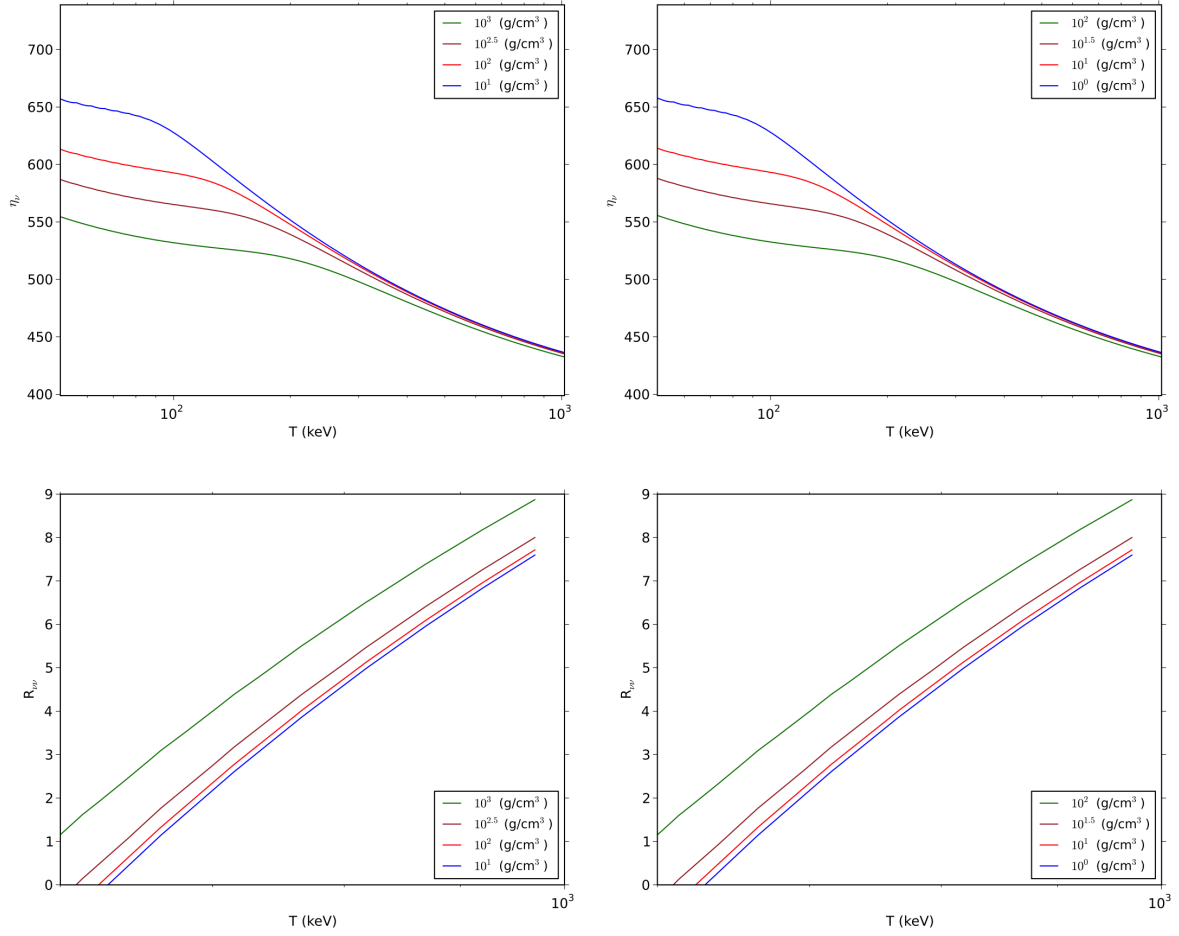


Fig. 12.— Contour plots of the critical value of baryon load parameter( $\eta_\nu$ ) (above) and number of expected events ( $R_{\nu\nu}$ ) (below) as a function of and temperature (T) (Meszaros & Rees 2000). We use the values of temperature and chemical potential in the resonance condition range and moderate field limit:  $10^{-4.3} B_c$  (left-hand figures) and  $10^{-5.3} B_c$  (right-hand figures). The number of events is calculated for  $z=0.1$